A REVIEW OF MATHEMATICAL MODELING OF CARBON POLLUTION BY VEHICLES ON HIGHWAYS WITH HUGE EMBANKMENTS

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Abstract

In this paper, we review developed mathematical models for carbon pollution and investigate the level of carbon pollutants on the highways and how it spreads to the people held up in traffic jams and those residing closer to the highways in Kenya.

INTRODUCTION

Exposure to high levels of diesel fumes containing black carbon has a threefold increased risk for contracting lung cancer compared with those exposed to low levels [1]. Highway traffic is one of the major sources of air pollution [2]. In [3] it is illustrated that ambient air concentrations of pollutants are frequently highest near highways. Since commercial and residential buildings often tend to be located near highways, it is important to have a satisfactory means of describing mathematically the expected pollutant concentrations near a specific highway under a given set of meteorological and vehicular conditions. Exhaust gases, which consists of black carbon, have very adverse effects on human health, besides negative environmental impact [4]. Revesz on page one of his journal 'the wall street' emphasized on the effect of carbon pollution by stating that the best way to move forward is to put price tag on carbon pollution that reflects its significant negative impact on climate and human health [5]. According to official statistical data by Teimuraz [6], air pollution in urban areas with heavy traffic is higher than in industrial areas. The same is now experienced in Kenyan capital cities namely Nairobi and Kisumu. It is expected that the continuous economic growth will strengthen the traffic intensity; therefore quality of air will worsen. When compounded with orography that prevents air ventilation, leads to pollutants density (concentration) grow in parts of Kenya, this includes highways with huge embankments. Thus, investigation of dispersion of exhaust gases in Kenyan highways, by means of mathematical modeling is very important for human health, environmental management and future economic planning, including revising of street network and traffic management where possible. The physical and chemical properties of carbon depend on the crystalline structure of the element. It can form three gaseous components with the oxygen namely; carbon monoxide, carbon dioxide, and carbon sub oxide, which are in symbol CO, CO_2, C_3O_2, respectively. Considerable technical developments have taken place in recent years with the emergence of advanced transportation systems, consequently strengthening these reactions on the environment.

The damages caused by carbon pollution in the world are immense [7]. Teimuraz [6] further emphasized that the main reason of the atmosphere air pollution is the exhaust of the motor-transport and exposure to traffic-related air pollutants that are linked to a wide variety of health concerns, including respiratory and cardiovascular problems, birth and developmental defects, and cancer. Given the enormous health and societal impacts resulting from near-road air pollution, it is critical to develop effective strategies to mitigate near-road carbon pollution [8]. In addition to vehicle emissions control, there are potential opportunities for mitigating near-road carbon pollution in roadway design options that affect pollutant transport and dispersion such as road configurations and the presence of roadside barriers. Recent wind tunnel and field studies have suggested that roadside barriers, such as sound walls and vegetation, may provide a cost effective strategy to mitigate near-road air pollution [9]. But it should also be noted that these barriers may affect the concentration of the pollutant. For instance, the discussion in [10] revealed that the presence of roadside barrier decreased concentration downwind or upwind but the flow also moved towards the barrier before being deflected upwards. In the same article it was further clarified that larger barriers led to an increased concentration on the roadway as there was more flow deceleration and therefore longer on-road residence time. In [11] it was observed that the presence of recitation zone had a significant impact on concentration. Due to barrier the flow recirculation occurs due to pressure difference. As the flow moves up and over the barrier, it concentrates creating a high-pressure region. This difference in pressure causes the flow to form circulation. Mathematical modeling of carbon pollution effectively enhances information about the air pollution level and provides a unique approach to basic knowledge in carbon pollution. It is an efficient tool for control of projects on air sanitation in areas with the individual extra protection. The model formulation process gives clarity on assumption and parameters. Based on this knowledge effective control management strategies can be employed. UNEP estimates that 90% of urban air pollution is rapidly growing in cities like Nairobi, and emirates from motor vehicles. Vehicle emissions are a mixture of particulate matter, carbon monoxide, sulfur oxides, nitrogen oxides and wide range of volatile non organic compounds.

PRELIMINARIES

In this section we present some basic definitions and useful terms that are important in the sequel.

Advection: transport of pollutants by bulk fluid flow

Aerosol: system of colloidal particles dispersed in a gas

Diffusion: movement of pollutants from a region where they are in high concentration to a region where they are in low concentration

Embankment: A levee, an artificial bank raised above the immediate surrounding land.

Orography: synonymous to oerography or orology or oreology; is part of a region's elevated terrain

Pollution: Introduction of contaminant in the natural environment that cause adverse change.

Pollutants: Any substance, certain chemicals or waste products, that renders air harmful.

Ultrafine: particulate matter of nanoscale size.

METHODOLOGY, RESULTS AND DISCUSSIONS

The advection-diffusion transport equation is complete with mass conservation and energy equations in this study and solved using the FVM. This is a discretization method for the approximation of a single or system of PDE's experiencing the conservation or balance of one or more quantities. These PDE's are often called conservation laws; they may be of different nature e.g. elliptic, hyperbolic and parabolic. They are used as models in a wide number of fields including chemistry, physics, biochemistry, biophysics etc. They describe the relationships between PDE's of unknown fields such as temperature, concentration, pressure, with respect to variables within the domain (space, time) under consideration. FVM have been constructed by dividing the continuous domain into a number of discrete sub-domains (control volumes) by grid. The grid defines the boundaries of the control volumes while the computational node lies at the center of the control volume. For each of the sub domains the governing algebraic equations are derived from the governing differential equation to obtain a system of algebraic equations. These equations are solved to obtain values of the dependent variables at the identified discrete points. Some of the advantages of the FVM are: FVM generally utilizes the divergence theorem to convert volume integrals in PDEs that contain a divergence term into surface integrals; it satisfies conservativeness and boundedness; it can be easily extended to complicated geometries with the aid of unstructured meshes; it is physically consistent with many engineering problems. We have found the solutions of the conservation laws using FVM, by employing Euler flux solvers, Godunov flux solvers, Riemann solvers, Intercellular flux solvers and Roe solvers for inversion. Since the model has diffusive term it has been simplified using Riemann solvers and Roe solvers to help in reducing the model to differential equation of first order.

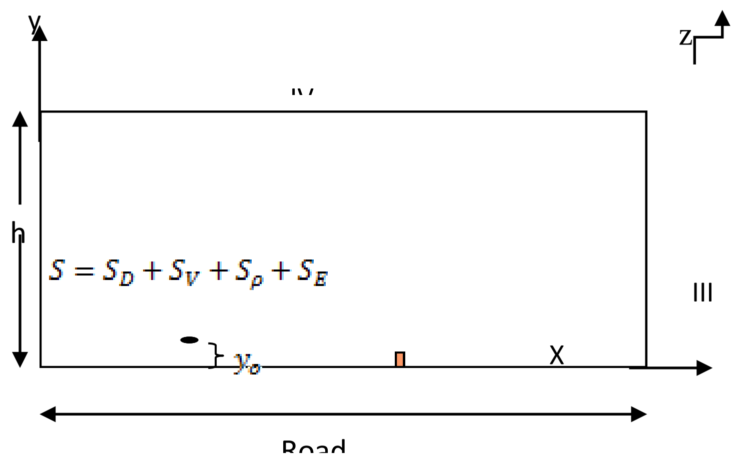

Fig 3 Geometric description of highway carbon pollution by

THE MODIFIED MODEL

In this study the Goyal and Anikender model is modified to incorporate embankment as a parameter in addition to making the flow unsteady, and the flow is uncompressible as opposed to compressible, see equations (7) and (8). The modified model incorporates more assumptions in addition to the ones in Goyal and Anikender model.

$$\frac{\partial c}{\partial t} + U\frac{\partial c}{\partial x} = \frac{\partial}{\partial x}\left(K_x \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial c}{\partial y}\right) S \ldots\ldots(7)$$

And the source term

$$S = S_D + S_V + S_\rho S_E \ldots\ldots(8)$$

Since the conservation of mass for incompressible fluids implies that
$\nabla . u0$..(9)

THE SOURCE TERMS

The Navier-Stokes equation for incompressible fluids is given by
$\rho \frac{Du}{Dt} = \rho g - \nabla p + \mu \nabla^2 \text{u}$.. (10)

Furthermore, the Euler equation arrived at when the viscosity is zero, in equation (10)$\mu = 0$:
$\frac{Du}{Dt} = g - \frac{1}{\rho} \nabla p$... (11)

When describing the dynamics of carbon pollution, it is necessary to include the source terms that control the emission of pollutants. This model implements four source terms all which take the form of
$\rho \frac{Du}{Dt} = \rho g \nabla . \tau$...(12)

DIFFUSIVE TERM, S_D

S_D acts to control the molecular diffusion between particles in the atmosphere
$S_D = \nabla . [D(x, y, t, \rho) \nabla \rho]$ ....(13)

For simplicity, we will assume the diffusive coefficient, $D(x, y, t, \rho)$ is a constant. This acts to greatly reduce the number of degrees of freedom for diffusion between molecules. The diffusion source term reduces to
$S_D D \nabla^2 \rho$..(14)

The last three source terms, $S_\rho, S_V, and\ S_E$ are associated with the emission of pollutants. Assume that the width of the emitter is x_0 and the height of the emitteris $y = y_o$. Assume that the emission of pollutant is time-independent. Let the mass density, ρ_e and velocity, V_{ey} of the emitted pollutant for y> y_o be expressed as $S_e = \rho_{eo} e^{\frac{y-y_o}{x_o}}$, V_{ey} =constant

SOURCE TERM ASSOCIATED WITH DENSITY, S_ρ

The source term associated with the emission of pollutants,S_ρ, consequently take the form

$$S_\rho \begin{cases} \frac{\dot{E}}{Ax_o} e^{\frac{y-y_o}{x_o}}, & y \geq y_o \\ 0, & y < y_o \end{cases} \qquad (15)$$

Where $\dot{E} = \rho_{eo} V_{ey} A$ is the flux of pollutants emitted from the area A. ρ_{eo} and V_{ey} are density and velocity of pollutants respectively.

SOURCE TERM ASSOCIATED WITH VELOCITY, S_V

This is given by
$S_V = S_\rho V_{ey} \hat{y}$..(16)

3.3.4 SOURCE TERM ASSOCIATED WITH ENERGY, S_E

The source associated with energy is given by

$$S_E = \frac{P_e}{\gamma-1}\frac{V_{ey}}{x_o} - \frac{1}{2}V^2(S_p + S_D) + V.S_V - \rho_V.g \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(17)$$

where, P_e is the pressure associated with emission of pollutant and $\gamma = \frac{C_P}{C_V} = 1.4$ is the adiabatic constant. Now that the source terms are known, the full hydrodynamic equations can be described.

THE HYDRODYNAMIC EQUATIONS

Carbon pollution dynamics can be described by the hydrodynamic equations (18) – (20)

$$\frac{\partial\rho}{\partial t} + \nabla.(\rho V^2) = S_\rho + S_D \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots... \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(18)$$

$$\frac{\partial(\rho\vec{V})}{\partial t} + \left[\left(\rho\vec{V}.\nabla\right)\vec{V}\right] = -\nabla P + \rho g + S_V \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots...\ldots\ldots\ldots\ldots\ldots\ldots...(19)$$

$$\frac{\partial E}{\partial t} + \nabla.\left(E\vec{V}\right) = -\nabla.\left(\mathrm{P}\vec{\mathrm{V}}\right) + \mathrm{S_E} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (20)$$

where P is gas pressure and $E = \frac{P}{\gamma-1} + \frac{\rho V^2}{2}$ is the total energy density, and internal kinetic.

Given that the viscous term is very small for large spatial scale, inviscid flow is assumed in the general set of the dynamic equations. The model is limited to two- dimensions as well. Now we need to discuss briefly each term in the conservation equations. Consider the mass conservation equation (18) which is a scalar equation.

$\frac{\partial\rho}{\partial t}$ is the time rate of change of mass density, which tells us how mass density changes with time $\nabla.\left(\rho\vec{V}\right)$, is a term associated with mass density and accounts for the mass density entering and leaving the system so that the flux in is equal to the flux out.

The source term S_ρ provides the emission of pollutants from the exhaust that we wish to analyze. The final term, the S_D source term accounts for the interaction between pollution particles through molecular diffusion.

We now consider the Navier-Stokes equation, (19) which is in vector form

$\frac{\partial(\rho\vec{V})}{\partial t} + \left[\left(\rho(\vec{V}.)\ \nabla\right)\ \vec{V}\right] = -\nabla P + \rho g + S_V$ It provides information about x and y spatial velocities since we assumed the two dimensional case. The first term $\frac{\partial(\rho\vec{V})}{\partial t}$ is the time rate of change of momentum that allows the velocity components V_x and V_y to be determined as a function of time. The next term $\left[\left(\rho\vec{V}.\nabla\right)\vec{V}\right]$ is the advection term, which accounts for the motion of pollutants through the atmosphere, in which the atmosphere is the bulk fluid that the pollutants move through. This advection term causes unwanted stabilities unless certain criterions are met.

The ∇P term is the pressure gradient which gives insight as to the path trajectory of the pollutants. This is because pollutants follow the decreasing pressure direction. The gravitational source term ρg accounts for the gravitational force on each pollutant. The final source term,$S_V$provides the source for the momentum of the pollution.

The final equation is the conservation of energy equation, (20).

$$\frac{\partial E}{\partial t} + \nabla.(E\vec{V}) = -\nabla.(P\vec{V}) + S_E$$

Given the system, equations (18) and (19) can be solved independent of the energy equation (20), therefore no need to discuss energy equation

GODUNOV'S METHOD

This is a conservative numerical scheme for solving PDEs. It is a conservative FVM which solves exact or approximate Riemann problems at each inter-cell boundary. Godunov scheme is used in non-linear system of hyperbolic conservation laws. The main analysis technique used to solve discontinuities in solution is to define cell averages by way of integration over the right and left-hand side of the discontinuous solutions.

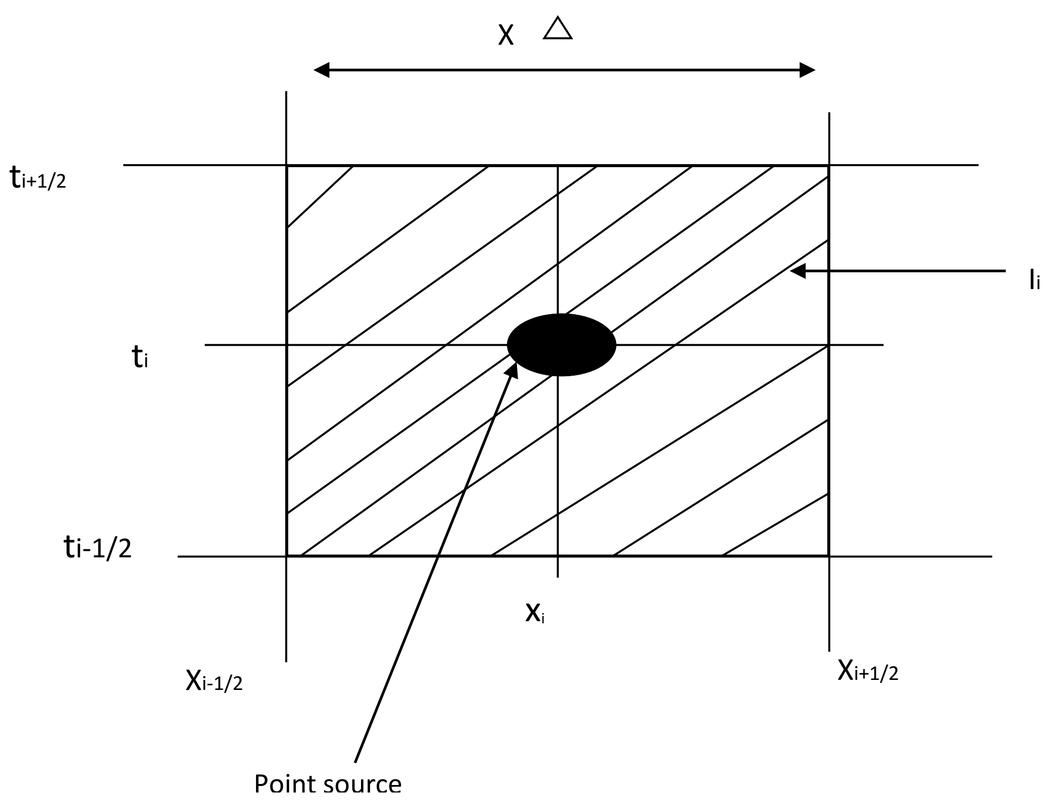

Fig 4

Godunov's first scheme is to define new average values U_i^{n+1} at time $t^{n+1} = t + \Delta t$by way of the integral

$U_i^{n+1} = \frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} \tilde{U}\,(x, t^{n+1})dx$Within each cell $I_i = \left[x_{i-1/2}, x_{i+1/2}\right]$

The above integral can be written as below

$$U_i^{n+1} = \frac{1}{\Delta x}\int_0^{\frac{1}{2}\Delta x} U_{i-1/2}\left(\frac{x}{\Delta t}\right)dx + \frac{1}{\Delta x}\int_{-\frac{1}{2}\Delta x}^{0} U_{i+1/2}\left(\frac{x}{\Delta t}\right)dx$$

The second more effective version of Godunov method can be written in conservation form as

$U_i^{n+1} = U_i^n + \frac{\Delta t}{\Delta x}\left[F_{i-1/2} - F_{i+1/2}\right]$ ……………………………………………………………(21)

With the inter-cell numerical flux given by $F_{i+1/2} = F\left(U_{i+1/2}, 0\right)$

If the time step Δt satisfies the stability condition

$\Delta t \le \frac{\Delta x}{S_{max}^n}$……………………………………………………………………………..(22)

Where S_{max}^n corresponds to the maximum wave velocity present throughout the domain at timet^n.

THE RIEMANN SOLVERS OF ROE

The Roe approximate solvers, is an approximate Riemann solver based on the Godunov scheme and involves finding an estimate for the inter-cell numerical flux or Godunov flux.

EXACT RIEMANN PROBLEM AND THE GODUNOV FLUX

A non-linear system of hyperbolic PDEs representing a set of conservation laws in one spatial dimension can be written in the form

$\frac{\partial U}{\partial t} + F\frac{\partial U}{\partial x} = 0OrU_t + F(U)_x$…………………………………………………………….(23)

With initial and boundary conditions

$$\begin{cases} U(x,0) = U_0(x) \\ U(0,t) = U_L(t) \\ U(L,t) = U_r(t) \end{cases}$$ ……………………………………………………………………….(24)

In the domain,$x_L \le x \le x_R$we will utilize Godunov's conservative formula, equation (21)

$U_i^{n+1} = U_i^n + \frac{\Delta t}{\Delta x}\left[F_{i-1/2} - F_{i+1/2}\right]$…………………………………………………………..(25)

The method of Roe to solve the Riemann problem will ultimately find solutions for the flux terms$F_{i-1/2}$and$F_{i+1/2}$so that the FVM can be utilized. The Godunov inter-cell numerical flux

$F_{i+1/2} = F\left(U_{i+1/2}(0)\right)$Was defined in chapter 3.5 and $U_{i+1/2}(0)$ is the exact similar solution $U_{i+1/2}\left(\frac{x}{t}\right)$ of the Riemann problem

$U_t + F(U)_x = 0$

$$U(x,0) = \begin{cases} U_L, & x < 0 \\ U_R, & x > 0 \end{cases}$$

which is evaluated at $\frac{x}{t} = 0$.

CONCLUSION

Embankment causes rapid increase in the concentration density of the pollutants within the highway closure to the ground; it also exaggerates effects of diffusion and advection of pollutants by limiting the effect of air flow in the solution domain. Godunov Riemann solver results compares well with the expected analytical results. Giving useful estimates of concentration of carbon pollutants spread from the source. Cancer incidences in Kenya are in an increasing trend, explanation of the cause to this is still unavailable. Therefore precautions are necessary and hence living near major roads with embankments and staying in traffic jam over long hours should be avoided to minimize possibility of development of asthma, impaired lung functions, birth and developmental effects and cardiovascular mortality. Therefore it has been clearly demonstrated that the presence of barriers has a detrimental effect on the air quality in road area but may help with more efficient dispersion downwind of the barrier.

In the next paper, we give a modified model, its stability conditions and its analysis.

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