INVARIANT SUBMANIFOLD OF $(3k,k)$ STRUCTURE MANIFOLD

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ABSTRACT

In this paper, we have studied various properties of a $(3k,k)$ structure manifold and its invariant submanifold, where k is positive integer. Under two different assumptions, the nature of induced structure ψ , has also been discussed.

Keywords : Invariant submanifold, Nijenhuis tensor, projection operators and complementary distributions.

1 Introduction

Let V^m be a C^∞ m-dimensional Riemannian manifold imbedded in a C^∞ n-dimensional Riemannian manifold M^n, where $m<n$. The imbedding being denoted by

$$f : V^m \longrightarrow M^n$$

Let B be the mapping induced by f i.e. $B=df$

$$df : T(V) \longrightarrow T(M)$$

Let $T(V,M)$ be the set of all vectors tangent to the submanifold $f(V)$. It is well known that

$$B : T(V) \longrightarrow T(V,M)$$

Is an isomorphism. The set of all vectors normal to $f(V)$ forms a vector bundle over $f(V)$, which we shall denote by $N(V,M)$. We call $N(V,M)$ the normal bundle of V^m. The vector bundle induced by f from $N(V,M)$ is denoted by $N(V)$. We denote by $C: N(V) \longrightarrow N(V,M)$ the natural isomorphism and by $\eta_s^r(V)$ the space of all C^∞ tensor fields of type (r,s) associated with N (V). Thus $\zeta_0^0(V) = \eta_0^0(V)$ is the space of all C^∞ functions defined on V^m while an element of $\eta_0^1(V)$ is a C^∞ vector field normal to V^m and an element of $\zeta_0^1(V)$ is a C^∞ vector field tangential to V^m.

Let $\bar{X}$ and $\bar{Y}$ be vector fields defined along $f(V)$ and $\tilde{X}, \tilde{Y}$ be the local extensions of $\bar{X}$ and $\bar{Y}$ respectively. Then $[\tilde{X}, \tilde{Y}]$ is a vector field tangential to M^n and its restriction $[\tilde{X}, \tilde{Y}]/f(V)$ to $f(V)$ is determined independently of the choice of these local extension $\tilde{X}$ and $\tilde{Y}$. Thus $[\bar{X}, \bar{Y}]$ is defined as

(1.1) $[\bar{X}, \bar{Y}] = [\tilde{X}, \tilde{Y}]/f(V)$

Since B is an isomorphism

(1.2) $[BX, BY] = B[X,Y]$ **for all** $X, Y \in \zeta_0^1(V)$

Let $\bar{G}$ be the Riemannain metric tensor of M^n, we define g and g^* on V^m and $N(V)$ respectively as

(1.3) $g(X_1, X_2) = \tilde{G}(BX_1, BX_2)f$, and

(1.4) $g^*(N_1, N_2) = \tilde{G}(CN_1, CN_2)$

For all $X_1, X_2 \in \zeta_0^1(V)$ and $N_1, N_2 \in \eta_0^1(V)$

It can be verified that g and g^* are the induced metrics on V^m and $N(V)$ respectively.

Let us suppose that M^n is a $(2k+S,S)$ structure manifold with structure tensor $\tilde{\psi}$ of type (1,1) satisfying

(1.5) $\tilde{\psi}^{3k}+\tilde{\psi}^{k}=0$

Let $\tilde{L}$ and $\tilde{M}$ be the complementary distributions corresponding to the projection operators

(1.6) $\tilde{l}=-\tilde{\psi}^{2k}, \qquad \tilde{m}=I+\tilde{\psi}^{2k}$

where I denotes the identity operator.

From (1.5) and (1.6), we have

(1.7) **(a)** $\tilde{l}+\tilde{m}=I$ (b) $\tilde{l}^2=\tilde{l}$ (c) $\tilde{m}^2=\tilde{m}$

(d) $\tilde{l}\,\tilde{m}=\tilde{m}\,\tilde{l}=0$

Let D_l and D_m be the subspaces inherited by complementary projection operators l and m respectively.

We define

$$D_l=\{X\in T_p(V):\ lX=X,\ mX=0\}$$

$$D_m=\{X\in T_p(V):\ mX=X,\ lX=0\}$$

Thus $T_p(V)=D_l+D_m$

Also $Ker\ l=\{X:lX=0\}=D_m$

$$Ker\ m=\{X:mX=0\}=D_l$$

at each point p of $f(V)$.

2. INVARIANT SUBMANIFOLD OF $(3k,k)$ STRUCTURE MANIFOLD

We call V^m to be invariant submanifold of M^n if the tangent space $T^p(f(V))$ of $f(V)$ is invariant by the linear mapping $\tilde{\psi}$ at each point p of $f(V)$. Thus

(2.1) $\tilde{\psi}BX = B\psi X$, for all $X \in \zeta_0^1(V)$,, and ψ being a (1,1) tensor field in V^m.

Theorem (2.1): Let $\tilde{N}$ and N be the Nijenhuis tensors determined by $\tilde{\psi}$ and ψ in M^n and V^m respectively, then

(2.2) $\tilde{N}(BX, BY) = BN(X,Y)$, for all $X, Y \in \zeta_0^1(V)$

Proof : We have, by using (1.2) and (2.1)

(2.3) $\tilde{N}(BX, BY) = [\tilde{\psi}BX, \tilde{\psi}BY] + \tilde{\psi}^2[BX, BY]$

$$-\tilde{\psi}[\tilde{\psi}BX, BY] - \tilde{\psi}[BX, \tilde{\psi}BY]$$

Simplifying the expression, we get (2.2),

3. DISTRIBUTION $\tilde{M}$ NEVER BEING TANGENTIAL TO $f(V)$

Theorem (3.1) if the distribution $\tilde{M}$ is never tangential to $f(V)$, then

(3.1) $\tilde{m}(BX) = 0$ for all $X \in \zeta_0^1(V)$

and the induced structure ψ on V^m satisfies

(3.2) $\psi^{2k} = -I$

Proof : if possible $\tilde{m}(BX)\neq 0$. From (2.1) We get

(3.3) $\tilde{\psi}^{2k}BX = B\psi^{2k}\ X$; from (1.6) and (3.3)

$$\tilde{m}(BX) = \left(I + \tilde{\psi}^{2k}\right)BX$$

$$= BX + B\psi^{2k}\ X$$

(3.4) $\tilde{m}(BX) = B\left[X + \psi^{2k}X\right]$

This relation shows that $\tilde{m}(BX)$ is tangential to $f(V)$ which contradicts the hypothesis. Thus $\tilde{m}(BX)$= 0. Using this result in (3.4) and remembering that B is an isomorphism, We get

(3.5) $\psi^{2k} = -I$

Theorem (3.2) Let $\tilde{M}$ be never tangential to $f(V)$, then

(3.6) $\underset{\tilde{m}}{\tilde{N}}(BX, BY) = 0$

Proof : We have

(3.7) $\underset{\tilde{m}}{\tilde{N}}(BX, BY) = [\tilde{m}\,BX, \tilde{m}BY] + \tilde{m}^2[BX, BY]$

$$-\tilde{m}[\tilde{m}BX, BY] - \tilde{m}[BX, \tilde{m}BY]$$

Using (1.2), (1.7) (c) and (3.1), we get (3.6).

Theorem (3.3) Let $\tilde{M}$ be never tangential to $f(V)$, then

(3.8) $\underset{\tilde{l}}{\tilde{N}}(BX, BY) = 0$

Proof : We have

(3.9) $\underset{\tilde{l}}{\tilde{N}}(BX, BY) = \left[\tilde{l}\, BX, \tilde{l}\, BY\right] + \tilde{l}^2 [BX, BY] - \tilde{l}\left[\tilde{l}\, BX, BY\right]$

$$-\tilde{l}\left[BX, \tilde{l}\, BY\right]$$

Using (1.2), (1.7) (a), (b) and (3.1) in (3.9); we get (3.8)

Theoren (3.4) Let $\tilde{M}$ be never tangential to $f(V)$. Define

(3.10) $\tilde{H}(\tilde{X},\tilde{Y}) = \tilde{N}(\tilde{X},\tilde{Y}) - \tilde{N}(\tilde{m}\tilde{X},\tilde{Y}) - \tilde{N}(\tilde{X},\tilde{m}\tilde{Y})$

$$+\tilde{N}(\tilde{m}\tilde{X},\tilde{m}\tilde{Y})$$

For all $\tilde{X},\tilde{Y} \in \zeta_0^1(M)$, then

(3.11) $\tilde{H}(BX, BY) = BN(X, Y)$

Proof : Using $\tilde{X} = BX$, $\tilde{Y} = BY$ and (2.2), (3.1) in (3.10) We get (3.11).

4. **DISTRIBUTION $\tilde{M}$ ALWAYS BEING TANGENTIAL TO $f(V)$**

Theorem (4.1) Let $\tilde{M}$ be always tangential to $f(V)$, then

(4.1) (a) $\tilde{m}(BX) = Bm\, X$ (b) $\tilde{l}(BX) = Bl\, X$

Proof : from (3.4), We get (4.1) (a). Also

(4.2) $l = -\psi^{2k}$

$$lX = -\psi^{2k}\, X$$

(4.3) $BlX = -B\psi^{2k}\, X$

Using (2.1) in (4.3)

(4.4) $BlX = -\tilde{\psi}^{2k} BX = \tilde{l}(BX)$,

which is (4.1) (b).

Theorem (4.2) Let $\tilde{M}$ be always tangential to $f(V)$, then l and m satisfy

(4.5) (a) $l + m = I$ (b) $lm = ml = 0$ (c) $l^2 = l$ (d) $m^2 = m$.

Proof : Using (1.7) and (4.1) We get the results.

Theorem (4.3) If $\tilde{M}$ is always tangential to $f(V)$, then

(4.6) $\psi^{3k} + \psi^{k} = 0$

Proof : From (2.1)

(4.7) $\tilde{\psi}^{3k} BX = B\psi^{3k} X$ Using (1.5) in (4.7)

$-\tilde{\psi}^{k} BX = B\psi^{3k} X$

$-B\psi^{k} X = B\psi^{3k} X$

Or $\psi^{3k} + \psi^{k} = 0$ which is (4.6)

Theorem (4.4) : If $\tilde{M}$ Is always tangential to $f(V)$ then as in (3.10)

(4.8) $\tilde{H}(BX, BY) = BH(X, Y)$

Proof: from (3.10) we get

(4.9) $\tilde{H}(BX, BY) = \tilde{N}(BX, BY) - \tilde{N}(\tilde{m}BX, BY) - \tilde{N}(BX, \tilde{m}BY) + \tilde{N}(\tilde{m}BX, \tilde{m}BY)$

Using (4.1) (a) and (2.2) in (4.9) we get (4.8).

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