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Department of Mathematics

Blood Flow in Large Femoral Artery

A Research Project Submitted in Partial Fulfillment of the Requirements

for the Master's Degree of Science in Mathematics

Presented to

Department of Mathematics
Faculty of Science, University of Tabuk

By

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Mariam Mohammad Hayaf

SUMMARY

The present master research project entitled, **Blood Flow in Large Femoral Artery**, has been spread over in three chapters. Chapter-1, gives the information and overall development of Bio-fluid Mechanics. This chapter further deals with basic definitions, governing equations of various models, different types of fluids and set of Magneto-hydrodynamic equations. In this chapter we have also discussed about the importance of the subject, cardiovascular system, distribution of blood volume and difference between anatomy and physiology.

The aim of the chapter-2 is based on the interest of today's research in Bio-fluid dynamics, more and more dedicated to accurate modeling of the blood flow characteristics. It became evident that blood is a complex fluid, with properties depending on many factors, not limited to shear rate and hematocrit. In this chapter we have discussed in details about the blood rheology and blood vessels because it is very important to understand the blood flow in the artery. Few properties of blood flow also have been discussed in this chapter.

Chapter-3, presents a theoretical model of oscillatory blood flow in circulatory rigid tube. The assumptions of a Newtonian rheology is considered and blood flow is described by the Navier-Stokes equations with specific term pressure gradient and boundary conditions. In this chapter we have got the analytical solution of the problem in terms of velocity and flow rate. The main aim of this chapter is, how the fluid dynamics play an important role in the understanding of the human circulatory system. This type of blood flow problem occurs in the human body in the case of diseased artery.

Committee Decision

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OVERALL DEVELOPMENT OF SUBJECT AND IMPORTANCE OF THE WORK

1.1 Introduction

Biological Science has been advanced from purely descriptive to analytical science. Many analytical methods of physical science have been used successively in the study of biological science.

1.2 Biofluid Mechanics

Bio-fluid mechanics is the study of certain class of biological problems from fluid mechanics. Bio-fluid mechanics does not involves any new development of the general principles of fluid mechanics but it involves some new applications of the method of fluid mechanics. the most common fluid mechanics problem in the biological system is the flow of blood. The number of basic variables in fluid mechanics is five: three velocity components and two thermodynamic properties. Hence we need five independent equations. These are usually the three components of the equation of motion, a continuity equation and an energy equation.

1.3 Laminar and Turbulent Flow

The terms laminar flow and purely viscous flow are used synonymously to mean a fluid flow which flows in laminae, as oppose to turbulent flow in which the velocity components have random fluctuations impose upon their mean values. Both laminar and turbulent flow occur in nature, but turbulent has been shown to exist in large arteries of a living system. It is especially pronounced when the flow rate increases in exercise conditions and to climb on the mountains.

1.4 Compressible and Incompressible Flow

we divide fluids into two groups- liquids and gases. Gases are compressible and their density changes readily with temperature and pressure. liquids, on the other hand, are rather difficult to compress and for all practical purpose may be considered as incompressible.

1.5 Basic Equations of Fluid Mechanics

1.5.1 Continuity Equation

$$\frac{\partial \rho}{\partial t} + \mathbf{q}.\nabla\rho + \rho \mathrm{div}\mathbf{q} = \mathbf{0} \qquad (1.1)$$

$$\frac{D\rho}{Dt} + \rho\text{div}\mathbf{q} = \mathbf{0}$$

Where the material derivative is

$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{q}.\nabla$, equation (1.1) is called the conservation of mass equation or the continuity equation. In the case of an incompressible fluid,

$$\frac{D\rho}{Dt} = 0 \tag{1.2}$$

hence the equation of continuity is, $\text{div}\mathbf{q} = \mathbf{0}$ (1.3)

that is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1.4}$$

where u, v, w denote the velocity components in the x, y, and z directions respectively.

1.5.2 Equation of Motion

$$\rho\frac{D\mathbf{q}}{Dt} = \rho\boldsymbol{F} - \nabla p + \mu\nabla^2\boldsymbol{q} \tag{1.5}$$

This is known as the Navier-Stokes equation for viscous, incompressible fluid. In Cartesian coordinate system this reduce to

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \text{v}\frac{\partial \text{u}}{\partial \text{y}} + \text{w}\frac{\partial \text{u}}{\partial \text{z}}\right) = \rho F_x - \frac{\partial p}{\partial x} + \mu\nabla^2 u \tag{1.6}$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + \text{v}\frac{\partial \text{v}}{\partial \text{y}} + \text{w}\frac{\partial \text{v}}{\partial \text{z}}\right) = \rho F_y - \frac{\partial p}{\partial y} + \mu\nabla^2 v \tag{1.7}$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + \text{v}\frac{\partial \text{w}}{\partial \text{y}} + \text{w}\frac{\partial \text{w}}{\partial \text{z}}\right) = \rho F_z - \frac{\partial p}{\partial z} + \mu\nabla^2 w \tag{1.8}$$

If however, the flow is frictionless then there are no shear stress and the normal stresses are just the pressure which is isotropic, in this case the equations of motion becomes

$$\rho\frac{D\mathbf{q}}{Dt} = \rho\boldsymbol{F} - \nabla p \tag{1.9}$$

These equations are known as the Euler equations for frictionless flow [8].

1.6 Basic Equations of Magneto-hydrodynamics (MHD)

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The complete set of magneto-hydrodynamic equations for a Newtonian, fluid flow includes the Navier- Stokes equations of motion, the equation of mass continuity, Maxwell's equations, and Ohm's law[3].

1.6.1 Navier-Stokes Equations with Lorentz Force

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}.\nabla)\mathbf{u}\right)=-\nabla p+\mu\nabla^2\mathbf{u}+\mathbf{j}\times\mathbf{B}+\rho\boldsymbol{g} \quad (1.10)$$

1.6.2 Equation of Continuity

$$\frac{\partial \rho}{\partial t}+\nabla.\rho\boldsymbol{u}=\mathbf{0} \quad (1.11)$$

1.6.3 Faraday's Law

$$\nabla\times\boldsymbol{E}=-\frac{\partial \boldsymbol{B}}{\partial t} \quad (1.12)$$

1.6.4 Ampere's Law

$$\nabla\times\boldsymbol{B}=\mu\boldsymbol{j} \quad (1.13)$$

1.6.5 Ohm's Law

$$\boldsymbol{j}=\sigma(\boldsymbol{E}+\boldsymbol{u}\times\boldsymbol{B}) \quad (1.14)$$

where the MHD body force $\mathbf{j}\times\mathbf{B}$ is included in the Navier- Stokes equation. The displacement current has been neglected from Ampere's law, which is a valid approximation for non-relativistic phenomena typical of the response of an inertial liquid. Implicit in equations from 1.10-1.14 are the following additional relations:

$$\nabla.\boldsymbol{B}=0 \quad (1.15)$$

$$\nabla.\boldsymbol{j}=0 \quad (1.16)$$

1.6.6 Magnetic Induction

$$\frac{\partial \boldsymbol{B}}{\partial t}=\nabla\times(\boldsymbol{u}\times\boldsymbol{B})+\frac{1}{\mu\sigma}\nabla^2\boldsymbol{B} \quad (1.17)$$

1.6.7 Vector Identity

$$\nabla\times(\nabla\times\boldsymbol{B})=\nabla(\nabla.\boldsymbol{B})-\nabla^2\boldsymbol{B} \quad (1.18)$$

1.7 Importance of the Subject

Biofluid dynamics is a fascinating subject due the fact it attracts attention from both the scientists and general public. we focus our attention on both the human body circulatory and respiratory systems. Understanding the circulatory system is one of the major areas of research and many engineers and scientists have carried out some remarkable work. The respiratory system is very closely linked to the circulatory system and is extremely complex to study and understand. The study bio-fluid dynamics is directed towards the finding solutions to some of human body related diseases and disorders.

Unlike engineering applications, to understand human body biofluid dynamics is not easy. This is due to the fact that in vivo experiments are not easy to perform. Non- invasive experiments are useful but not always give the desired result. Thus, both theoretical and computational biofluid dynamics play a major role in the understanding of human body biofluid dynamics. Thus, in the development of medical devices both theoretical and computational biofluid dynamics play an important role at the early stages of design and development [12].

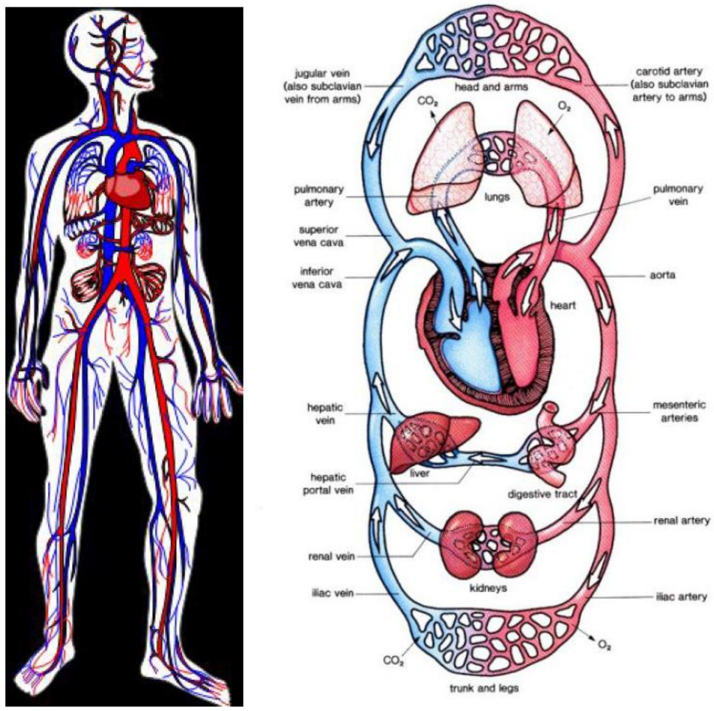
1.8 The Role of Mathematics in Biology

In fact, one role of mathematics interested in biological and problems is to evolve new mathematical methods for dealing with complex situations in life sciences. There are large areas of bio sciences, which are not yet amenable to mathematical treatment, however mathematical bio science constantly endeavor to widen the areas to which mathematical techniques can be applied for gaining a better insight and deep knowledge, our understanding of those areas, which have already been mathematiczed.

1.9 The Cardiovascular System

The discovery of the circulation of blood in the human body is related to William Harvey (1578-1657). The cardiovascular system or circulatory system consists of the heart (pump) and a network of tubes that transport the blood. The circulatory Transport system is responsible for oxygen and nutrient supply to all body tissue and removal of waste products. The heart consists of two pulsatile pumps in series and circulates blood through the vasculature (network, see fig.1-a). The vasculature consists of arteries, arterioles , capillaries , venules and veins (see, fig.1-b).

Hence there must be adequate circulation at all times to the important organs of the body- brain, heart, and lungs. Should circulation fail or malfunction, various diseases and even death could occur. Thus, the importance of the circulatory system cannot be overemphasized.



(a) Whole system (b) Simplified system

Figure 1-a,b: Schematic description of the circulatory system

The cardiovascular system circulates about 5 liters of blood at a rate of approximately 6 liters per minute . The blood travels continuously through two separate loops; both originate and terminate at the heart. The pulmonary circulation carries blood between the heart and the lungs, whereas the systemic circulation carries blood between the heart and all the organs and body tissues (fig.1-c). In both system blood is transported in the vascular bed because of a pressure gradient through the arteries, arterioles, capillaries, venules, and veins. the cardiac cycle is composed of the diastole, during which the ventricles are filling with blood, and the systole, during which the ventricles are actively contracting and pumping blood of the heart [9] .

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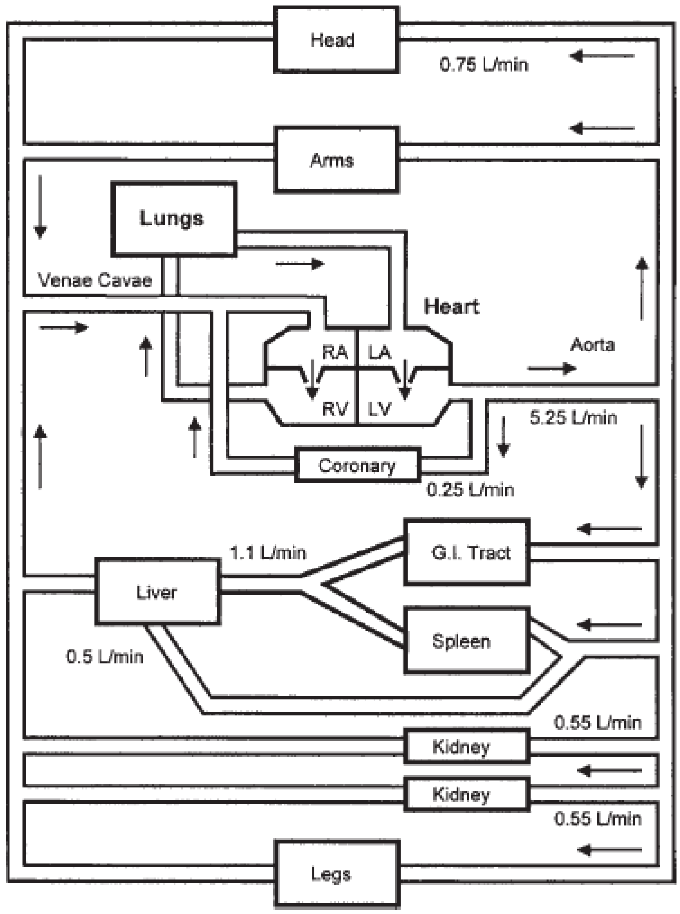

Fig.1-c: General organization of the circulatory system with averaged values of normal blood flow to major organs.

1.10 Distribution of Blood Volume in the Various Components of Circulatory System

The total volume is unevenly distributed as follows: about 100% of the entire blood volume is in the systemic circulation, with 64% in the veins, 13% in the arteries system, and 7% in the arterioles and capillaries. The heart contains 7% of blood volume and the pulmonary vessels 9%. (see table1-1)

Table-1.1 Arterial Structure

Structure	Percentage of Total Blood Volume
Systemic venous system	64
Systemic arterial system	13
Capillaries	7
Pulmonary circuit	9
Heart	7

1.11 Blood Flow Distribution in Cardiovascular System

Under the normal conditions the distribution of blood flow to the various organs is brain 13%, heart 5%, kidneys 25%, liver and gut 20%, skeletal muscles 20%, skin 7%, and others10% (Fig:1-d). At normal resting activities heart rate of an adult is about 75 beats/min with a stroke volume of typically 70 mL/beat. The cardiac output, the amount of blood pumped each minute, is 5.25 L/min. It decline with age. During intense exercise, heart rate may increase to 150 beats/min and stroke volume to 130 mL/beat, providing a cardiac output of about 20 L/minute. [25]

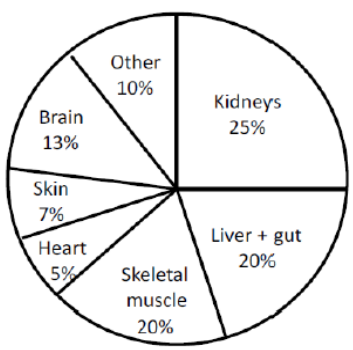

Figure: 1-d, Percentage wise distribution of blood in different parts of the body.

1.12 Difference Between Anatomy and Physiology

1.12.1 Anatomy

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Before going on chapter-2 we shall know about the anatomy and physiology. anatomy is the study of the internal and external structures of the plants, animals or the human body. For example, the study of the arrangement of the bones that comprise the human skeleton, which is the anatomical from work for our bodies, is considered anatomy [14].

1.12.2 Physiology

Physiology focuses on the function and vital process for the various structures making up the human body. These physiological process includes muscle contraction, our sense of smell and sight, how we breath and so on.

In other words, anatomy focuses on the structure and how something is put together, whereas physiology is the study of how these different structures work together to make the body function as a whole. For example, anatomy would be the study of structure of the red blood cells (R.B.Cs), and physiology would be the study of how the R.B.Cs carry vital oxygen thought our body.

1.12.3 Mathematical physiology

This deals with mathematical models of the brain, conduction of current in the nerve cells, exchange of oxygen and carbon dioxide in the human system, and the functioning of various sense organs and kidneys. This analyzes the uses of mathematical methods in the diagnosis and treatment of specific diseases such as cancer [14].

1.12.4 Mathematical Biomechanics

This deals with, (a) bio-solid mechanics which studies among other things, stresses and strains in bones and muscles, (b) bio-fluid dynamics which includes a study of the flow of blood in arteries and veins, (c) cochlear dynamics which is mainly concerned with the flow of fluids in the inner ear, and (d) lung dynamics which is primarily a study about the flow of gases in lung airways.

1.12.5 Mathematical Bioengineering

This involves the design of medical appliances such as delayers, heart, lung mechanics, artificial limbs, and computerized axial topographic scanners [10,12].

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CHAPTER -2

BLOOD RHEOLOGY AND BLOOD VESSELS

2.1 Introduction

The fluids associated with human body includes air, oxygen, carbon dioxide , water, solvents, suspensions, serum, lymph, and blood. This chapter gives importance to the major body fluid, blood. This is mainly due to the fact that the blood is extremely complex fluid. it consists of blood cells suspended in plasma. since blood flow in arteries and veins are closely linked to the blood vessels properties, morphology of blood vessels will also be discussed in this chapter.

The major functions of the blood include:

1. Carries oxygen and nutrients to active tissues.

2. Delivers carbon dioxide to the lungs.

3. Brings metabolic end products to kidneys.

4. Blood is a buffering reservoir that controls pH of biofluids.

5. It plays a major role in the body's immune system.

6. In addition to mass, it also regulates the body temperature by moving core heat to the periphery, where it can be dissipated into the external environment. To study the blood blow, it is essential to learn about its composition [6].

2.2 Blood Components

2.2.1 Blood Plasma

Blood plasma is a liquid component of blood, in which the blood cells are suspended. it makes up about 55% of total blood volume. It is composed of mostly water, and contains dissolved proteins, glucose, clotting factors, mineral ions, hormones and carbon dioxide. All the major components of blood plasma are given in table 2.1. in addition to the components which are given in the table, plasma also contains small quantities of gasses, carbon dioxide (2ml/100), oxygen (0.2ml/100ml) and nitrogen (0.9ml/100ml), composition of nutrients such as amino acids (40mg/100ml) and vitamins (0.0001-2.5 mg/100ml), and the composition of waste products such as urea (34mg/100ml), creatinite (1mg/100ml), uric acid (5mg/100ml), and bilirubin.

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The density of plasma is approximately 1030 kg/m^3. with natural proteins the plasma behaves like a Newtonian fluid with a viscosity of 1.2 × 10^{-3} Poise. The inorganic components in the plasma generates an osmotic pressure of about 8× 10^5 Pa.

Table 2.1: Composition of blood plasma

Component	Percentage/Concentration	Functions
1. Water	93%	Carrying medium
2. Proteins	6-8%	buffer; binding agent; antibodies Clotting factor; enzymes; non-penetrating solute
2.1 Albumin	3.3 - 4.0 g/100 ml	
2.2 Globulins	≈ 4.5 g/100 ml	
2.3 Fibrinogen	0.34-0.43 g/100 ml	
3. Inorganic components	0.8%	
3.1 Sodium	0.31-0.34 g/100 ml	
3.2 Potassium	0.016-0.021 g/100 ml	
3.3 Calcium	0.009-0.011 g/100 ml	
3.4 Magnesium	0.002-0.003 g/100 ml	
3.5 Chloride	0.36-0.39 g/100 ml	
3.6 Bicarbonate	0.20-0.24 g/100 ml	
3.7 Phosphate	0.003-0.004 g/100 ml	
4. Lipids	0.6%	
5. Glucose	0.1%	

2.2.2 Blood Cells

Blood cells are normally divided in to three components, red blood cells (erythrocytes), white blood cells (leukocytes) and platelets. The white blood cells and platelets play the important role in the immune response and blood clotting. however, the number of white blood cells and platelets are relatively small as compared to the number of red blood cells.

Table 2.2: Blood cells

Cell	Volume in blood, %	Number per mm^3	Unstressed shape and size, (μm)
Erythrocytes	45	4-6×10^6	Biconcave disc, 8 × 1-3
Leucocytes		4-11×10^3	Roughly sphereical, 7-22
Platelets	1	2.5-5×10^5	rounded or oval, 2-4

2.2.3 Erythrocytes

The word erythrocyte comes from the Greek erythros which means "red" and kytos it means "hollow" which is commonly known as "cell". As shown in table 2.2, erythrocytes volume

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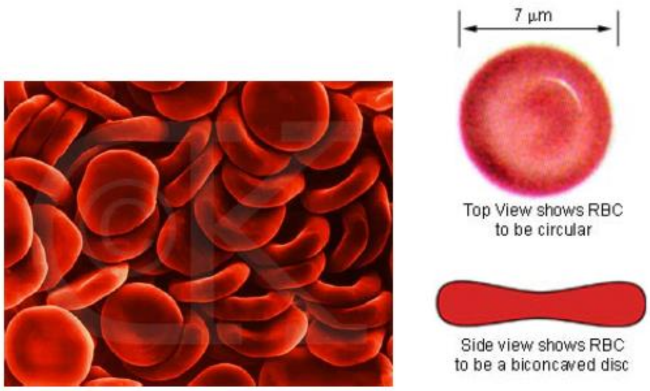
concentration in blood is about 45%. This concentration is often known as haematocrite (the ratio between the volume of red blood cells to the total blood volume). The diameter of the disc is about 7µm and the thickness varies between 1 and 3 µm. The density of red blood cells1.08×10^{3}. The life span of red blood cells is approximately 125 days. This means that about 0.8% of all red blood cells are destroyed each day, while the same amount of red blood cells are produced in the bone marrow. Iron is required to produce the hemoglobin necessary for red blood cell production and 1 to 4 mg of iron per day is the minimum requirement. The liquid interior is a saturated solution of hemoglobin with a dynamics viscosity of 6×10^{-3}. The shape of red blood cell is shown in figure (2.1)



(a) Group (b) Single

Figure 2.1: Red blood cells (erythrocytes)

2.2.4 Leukocytes

They are very small numbers of as compared to red blood cells. Thus they have very small influence on the rheological properties of blood. White blood cells can be defined in to two groups, arranged by function as phagocytes and immunocytes . They can also be classified into two groups by appearance as granulocytes and a granulocytes. Healthy whole blood normally contains approximately 4,000 to 11,000 leukocytes in each cubic millimeter. The white blood cells play a major role in fighting diseases. Table 2.3 includes list of types of leukocytes [25].

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Table- 2.3 Various Types of Leukocytes Grouped in Order of Their Relative Numbers.

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Name	Count per cubic millimeter	Size, µm
Neutrophil	2500–7500	10–15
Lymphocyte	1000–3000	10–20
Monocyte	200–800	20–25
Eosinophil	40–400	10–15
Basophil	10–100	10–12
Total leukocytes	4000–11,000	

2.2.5 Platelets or thrombocytes

The volume concentration of platelets is about 0.3%. They are more rigid than red blood cells and responsible for blood clots. If the platelets are in contact with adenosine diphosphate (ADP) they aggregate and thrombus forms. The mean diameter of a platelet is about 1 to 2 µm or about one eighth the diameter of erythrocyte. The normal life span of a thrombocyte is 7 to 10 days. The normal platelets count in healthy humans is about 250,000 per cubic millimeter of whole blood. Platelets are granular in appearance and have mitochondria but no nucleus. The main function of platelets is the formation of mechanical plugs during normal haemostatic response to vascular injury.

2.2.6 Blood pH

The pH of normal healthy blood is in the range of 7.35 to 7.45. when the pH is less than 7.35, this condition is defined as acidosis. When the pH is greater than 7.45, this condition is defined as alkalosis. Co_2 dissolved in water in plasma produces carbonic acid, which lowers blood pH. Bicarbonate and carbonic acid form an acid-base buffer pair, which helps to keep the arterial pH near 7.4.

2.3 Blood Vessels

Arteries are the high pressure blood vessels that transport blood from the heart, through increasingly smaller arteries, to arterioles, and further to the level of capillaries. Veins conduct the blood from the capillaries back to the heart on the lower pressure side of the cardiovascular system. At any given time about 13% of the total blood volume resides in the arteries, and about 7% resides in the capillaries. Veins, arteries, and capillaries differ in structure because they are specialized to perform their respective perfusion, exchange, and capacitance function. However, the inner layer of all blood vessels is lined with a single layer of endothelial cells [11].

2.3.1 General Structure of Arteries

There are three types of arteries which can be classified according to their size and structure. **Elastic arteries**, which include the aorta, have a relatively greater diameter and a greater number of elastic fibers. **Muscular arteries** are smaller in diameter than elastic arteries, but larger than arterioles, and they have a relatively larger proportion of muscle compared to connective tissue. **Arterioles** are the smallest diameter arteries and have a few layers of smooth muscle tissue and almost no connective tissue. In mathematical or physical terms, the vessels are not rigid but elastic.

In general, arteries are composed of three layers: the **tunica intima** or the innermost layer, the **tunica media** or middle layer, and the **tunica externa**, which is the outermost layer of the artery [15] The radius size of ascending aorta (Large Artery) and Arteriole (Small artery) is about to 1.25 and 0.01 cm respectively.

The three layers of an artery are shown in fig. (2.2)

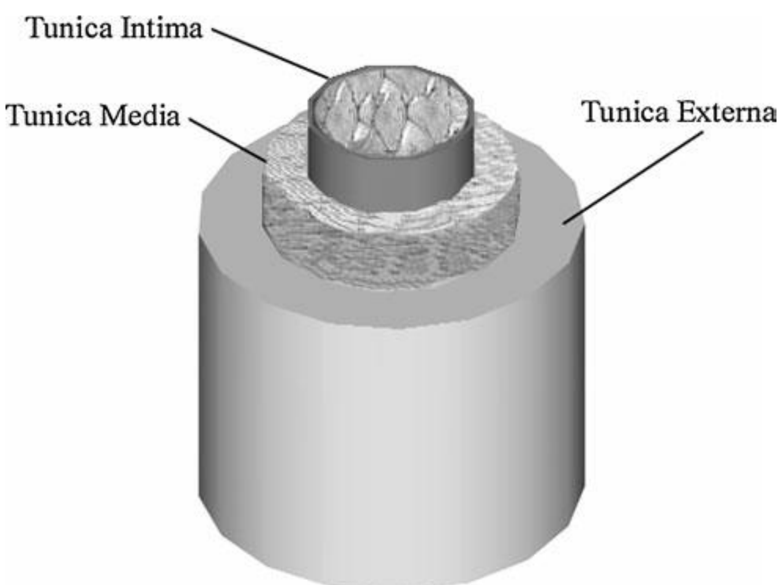

Fig. 2.2 Structure of an artery

The lumen of an artery is the inside of the vessel where blood flows. The lumen is lined with the endothelium which forms the interface between the blood and the vessel wall. In tunica intima

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the endothelium plays an important role in the mechanics of blood flow, blood clotting, and leukocyte adhesion.

The tunica media consists primarily of smooth muscle cells. The tunic media is living and active. The tunica media can contract or expand and change diameters of the vessels, allowing a change in blood flow. vasoconstriction is the word used to describe a reduction in vessel diameter due to muscular contraction. When smooth muscle in the tunica media relaxes the diameter of the vessel increases at a given pressure. This increase allows more blood flow for the same deriving pressure and this process is known as vasodilation.

The tunica externa is also sometimes called the tunica adventitia. The tunica externa is composed of connective tissue including passive elastic fibers as in the tunica media. Just like the tunica media, the tunica externa is absent in capillaries[9].

2.4 Blood Rheology

Rheology is the study of the deformation and flow of matter. Thus, it is important to know about the blood rheology. Rheology explains how a material deform /flow in response to applied forces. If the blood is stationary for several seconds, rouleaux begin to form. As a result the effective viscosity of the blood increases. When stationary state is disturbed with increasing shear rate, the rouleaux formation is destroyed and the viscosity decreases. The red blood cells in the body are often influenced by two mechanisms. First one is the random Brownian motion attempting to create a random aggregation of red blood cells. The second mechanism in which the flow attempts to orient the red blood cells in the streamline direction with the highest axis aligned with streamline direction. The former mechanism increases the effective viscosity, while the later one decreases the viscosity. From these physics, it is clear that blood is a thinning fluid i.e., viscosity decreases with increases in shear rate [10].

2.4.1 Viscosity

The material property that is represented by the slope of the stress-shearing rate curves is known as viscosity and is represented by the Greek letter μ. Sometimes it is also known as absolute viscosity or dynamic viscosity. Kinematic viscosity is a another fluid property that has been used to characterize flow. It is the ratio of absolute viscosity to fluid density and is denoted by the Greek letter ν (nu), that is $\nu = \frac{\mu}{\rho}$. The SI units for absolute viscosity and kinematic viscosity are Ns/m^2 and m^2/s respectively. The viscosity of a liquid decreases rapidly with increasing temperature whereas the viscosity of gas increases with temperature. The viscosity of fluids also depends on pressure, but this dependence is usually of little importance compared to the temperature variation in problems of fluid dynamics.

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2.4.2 Viscosity of blood

Viscosity is strongly dependent on temperature, but in humans are maintained at a constant 37^{o}C, the issue of viscosity change with temperature is not significant. On the other hand, blood viscosity varies with such parameters as hematocrit, shear rate, and even vessels diameter. For shear rates above 100 s^{-1}, viscosity becomes constant, that is why at these shear rate blood behaves a Newtonian fluid [12].

2.5 Types of fluids

2.5.1 Newtonian Fluids

For fluids with constant viscosity are known as Newtonian fluids. For example, fluids like oil, water, and air, viscosity does not vary with shearing rate. For Newtonian fluids, shear stress and rate of shearing strain be related to the following equation:

$$\tau = \mu_e \, \dot{\gamma} \qquad (2.1)$$

Where τ = Shear stress, μ_e = viscosity, $\dot{\gamma}$ = the rate of shearing strain. Relation (2.1) is known as Newton's law of viscosity. For a plot shown in fig. 2.3.

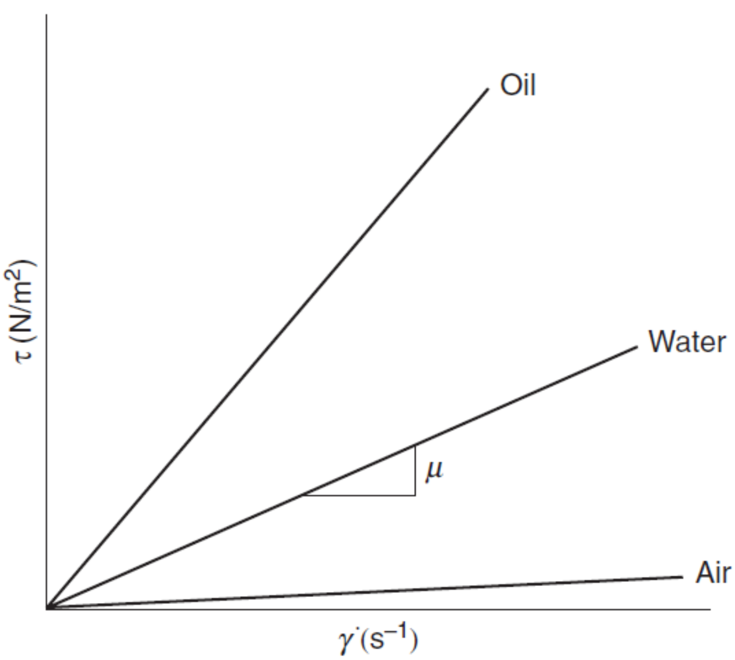

Fig. 2.3 Stress versus rate of shearing strain for various fluids.

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2.5.2 Non-Newtonian Fluids

For non-Newtonian fluids, τ and $\dot{\gamma}$ are not linearly related. Therefore, the slope of the shear stress/shear rate curve is not constant. For a plot shown in fig.(2.4), for those fluids, viscosity can change as a function of the shear rate. For examples, salt solutions, molten, ketchup, custard, toothpaste, starch suspensions, paint and shampoo, enamels, varnish, wet clay, mud, emulsion of oil in water etc. Blood is also an important example of a non-Newtonian fluid.

(a) Shear Thinning or Pseudo-plastic Fluids

The most common type of time-independent non- Newtonian fluids are whose apparent viscosity decreases as shearing rate increases. Latex, paint is the good examples of shearing thinning fluid. At low (<100 s^{-1}) shear rates, blood behaves like a shear thinning fluid. However, when the shear rate increases above 100 s^{-1}, blood behaves as a Newtonian fluid.

(b) Shear Thickening Fluids

Shear thickening fluids are non-Newtonian fluids whose apparent viscosity increases when the shear rate increases. Quicksand is a good example of shear thickening fluid. A mixture of cornstarch and water also forms a shear thickening fluid.

(c) Bingham Plastic

A Bingham plastic is neither a fluid nor a solid. A Bingham plastic can withstand a finite shear load and flow like a fluid when that shear stress exceeded. Toothpaste and mayonnaise are examples of Bingham plastics. Blood is also Bingham plastic and behaves as a solid at very low shear rate that is close to zero. The yield stress for blood is very small approximately from 0.005 to 0.01N/m^2 .

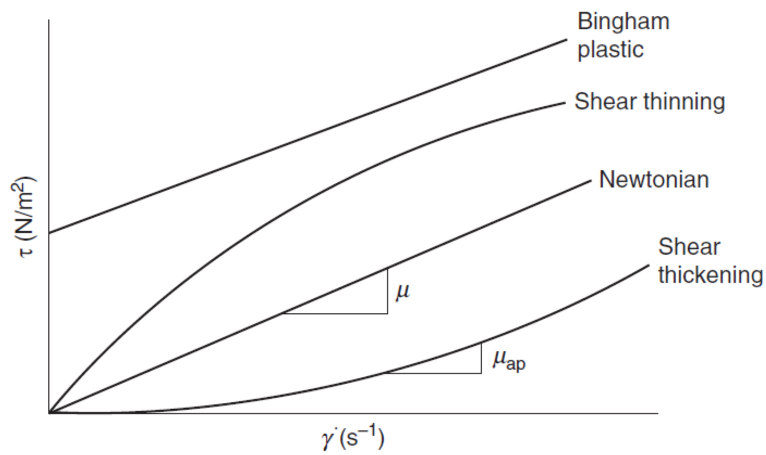

Fig. 2.4 Stress versus rate of shearing strain for various for some non- Newtonian fluids

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2.6 Blood Constitutive Models

2.6.1 Time Independent Fluids

For a time independent fluid non-Newtonian fluid, the constitutive equation is

$$\tau = f(\dot{\gamma}) \qquad (2.2)$$

A Newtonian fluid is just a special case of non-Newtonian fluid where the function $f(\dot{\gamma})$ is linear. See fig. 2.5 shows typical shear stress strain rate relation for non- Newtonian fluids.

2.6.2 Power Law Model

one important class of non-Newtonian fluids is that of power law fluids which have constitutive equation

$$\tau = \mu\dot{\gamma} = \mu\dot{\gamma}^{n-1}\dot{\gamma} \qquad (2.3)$$

This class of non-Newtonian fluids has effective viscosity coefficient or apparent viscosity $\mu\dot{\gamma}^{n-1}$ and does not have a yield stress.

For n<1, the fluid exhibits shear thinning properties, we get a pseudo plastic power law fluid characterized by a progressively decreasing apparent viscosity with strain rate.

For n=1, we obtain the Newtonian fluid as special case.

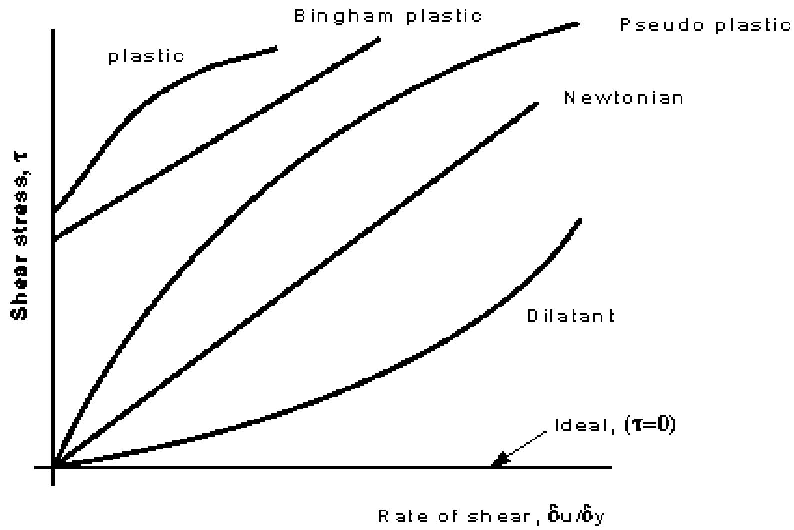

Fig.2.5 Shear stress strain rate relationship for non-Newtonian fluid.

For n>1, the fluid shows shear thickening behavior and we have dilatants power law fluid in which the apparent viscosity increases with increasing strain rate.

Several objection are raised against the power law model, one of them being for a pseudo plastic fluid, the apparent viscosity is infinite when the shear strain rate is zero. However, this model has been found desirable because of its simplicity, as well as to analyze such flows as Couette flow in pipes an channels.

2.6.3 Bingham Model

Another important fluid is the Bingham fluid, has constitutive equation

$$\tau = \mu\dot{\gamma} + \tau_0, \quad \tau \geq \tau_0$$

$$\dot{\gamma} = 0, \qquad \tau \leq \tau_0 \tag{2.4}$$

2.6.4 Herschel-Bulkley Model

$$\tau = \mu\dot{\gamma}^n + \tau_0, \quad \tau \geq \tau_0 \tag{2.5}$$

$$\dot{\gamma} = 0, \qquad \tau \leq \tau_0$$

2.6.5 Casson Model

$$\sqrt{\tau} = (\mu\dot{\gamma})^{\frac{1}{2}} + (\tau_0)^{\frac{1}{2}}, \qquad \tau \geq \tau_0 \tag{2.6}$$

$$\dot{\gamma} = 0, \qquad \tau \leq \tau_0 \tag{2.7}$$

2.7 Time Dependent Fluids

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Some fluids are more complex than those just described and the apparent viscosity depends not only on strain rate but also on the time. These can generally be classified into two classes:

(i) Thixotropic fluids (ii) Rheoplastic fluids.

For thixotropic fluid, the shear stress decreases with time as the fluid is sheared, while for a rheoplastic fluid, the shear stress increases with time as the fluid is sheared. Examples of a thoxotropic fluid is printer's ink, Non-drip pains, wet cement, waxy crude oils etc.

2.8 Cardiovascular Diseases

Cardiovascular diseases refer to the class of diseases that involve the heart or blood vessels. [2]

2.8.1 Aneurysms

An aneurysm is a localized, blood-filled dilation of a blood vessel wall. Most common areas prone to aneurysms in arteries are at the base of brain and in the aorta. The aneurysms of brain are referred as cerebral aneurysms. The aortic aneurysms can be classified based on the locations. One common form of the thoracic (aortic) aneurysm involves winding of the proximal aorta and the aortic root. The aneurysms at the distal part of the aorta are known as abdominal aortic aneurysms. Aneurysms occur in the legs also, particularly in the deep vessels such as the popliteal vessels in the knee. The bulge in a blood vessels can burst and lead to death at any time. Since the aneurysms have natural tendency to grow, given enough time they will inevitably reach the bursting point if undetected.

2.8.2 Angina

Angina pectoris, commonly known as angina, is severe chest pain due to ischemia (lack of blood and oxygen supply) of the heart muscle. This happens generally due to obstruction or spasm of the coronary arteries that supply blood to heart walls. Coronary artery disease, the main cause of angina, is due to atherosclerosis of the cardiac arteries.

2.8.3 Atherosclerosis

Atherosclerosis is a disease affecting arterial blood vessels. It is a chronic inflammatory response in the walls of arteries, in large part due to the accumulation of macrophage white blood cells and promoted by low density lipoproteins (LDL) without adequate removal of fats and cholesterol from the macrophages by functional high density lipoproteins (HDL). It is commonly referred to as **hardening** or **furring** of arteries. It is caused by the formation of multiple plaques within the arteries.

These complications are chronic, slowly progressing and cumulative. Most commonly, soft plaque suddenly ruptures, causing the formation of a thrombus that will rapidly slow or stop blood flow, leading to death of the tissues fed by the artery in approximately 5 minutes. This catastrophic event is called an infraction or myocardial infarction (a heart attack). Another common scenario in very advanced disease is claudication from insufficient blood supply to the legs, typically due to a combination of both stenosis and aneurysmal segments narrowed with clots. Since atherosclerosis is a body wide process, similar events occur in the arteries to the brain, intestines, kidneys, legs, etc.

2.8.4 Stenosis

Stenosis refers to an obstruction of flow through a vessel. When a localized plaque forms inside a vessel, this is called stenosis. An aortic stenosis refers to an obstruction at the level of the aortic valve. An aortic stenosis is typically seen as a restricted systolic opening of the valve with an increased pressure drop across the valve.

2.8.5 Stroke

A stroke is the rapidly developing loss of brain functions due to a disturbance in the blood vessels supply to the brain. This can be due to ischemia caused by thrombosis or embolism or due to hemorrhage. As a result, the affected area of the brain is unable to function, leading inability to move or more limbs on one side of the body, inability to understand or formulate speech or inability to see one side of the visual field.

2.8.6 Cerebrovascular Disease

Cerebrovascular disease is a group of brain dysfunctions related to disease of blood vessels supplying the brain. Hypertension is the most important cause that damages the blood vessel lining endothelium exposing the underlying collagen where platelets aggregate to initiate a repairing process which is not always complete and perfect. Sustained hypertension permanently changes the architecture of the blood vessels making them narrow, stiff, deformed and uneven which are more vulnerable to fluctuations of blood pressure. A fall in blood pressure during sleep can lead to marked reduction in blood flow in the narrowed blood vessels causing ischemic stroke in the intracranial hemorrhage during excitation at daytime. Primarily people who are elderly, diabetic, smoker, or have ischemic heart disease, have cerebrovascular disease. This is simplistic study by which arteries are blocked by fatty deposits or by a blood clot. The results of cerebrovascular disease can include stroke, or even sometimes a hemorrhage stroke.

2.8.7 Heart Failure

Heart failure is a cardiac condition that occurs when a problem with the structure or function of the heart impairs its ability to supply sufficient blood flow to meet the body's needs.

2.9 Respiratory Diseases

Respiratory disease is the term for diseases of the respiratory system. These include diseases of the lung, pleural cavity, bronchial tubes, trachea, upper respiratory tract and of the nerves and muscles of breathing. Respiratory diseases range from mild and self-limiting such as the common cold to life threatening such as bacterial pneumonia, tuberculosis, swine flu and asthma. The study of respiratory system is known as pulmonology. A doctor who specializes in respiratory diseases is known as pulmonologist, or a chest medicine specialist.

2.9.1 Chronic Obstruction Pulmonary Disease

Chronic obstruction pulmonary disease **(COPD),** also known as chronic obstruction airways disease, is group of illness characterized by airflow limitation that is not fully reversible. The flow of air into and out of lungs is impaired. The term COPD includes the conditions emphysema and chronic bronchitis. In many parts of the world, the most common cause of obstructive lung disease is lung scarring after tuberculosis. The most common cause of COPD is cigarette smoking. COPD is gradually progressive condition and usually only develops after about 20 pack years of smoking.

2.9.2 Asthma

Asthma is an obstructive lung disease where the bronchial tubes (airways) are extra sensitive. The airways become inflamed and produce excess mucus and the muscles around the airways tighten making the airways narrower. Asthma is usually triggered by breathing in things in the air such as dust or pollen that produce a allergic reaction. It may be triggered by other things such as an upper respiratory tract infection, cold air, exercise or smoke.

CHAPTER-3

OSCILLATORY BLOOD FLOW IN A RIGID CIRCULAR FEMORAL ARTERY

3.1 Introduction

Blood as fluid that human and other living creatures are dependent on has been always considered by scientists and researchers. Any changes in blood pressure and its normal velocity can be a sign of a disease. The arteries supplying blood, must respond to local changes in the end organs while maintaining overall homeostasis of the circulatory system. Biofluid mechanics describe the kinematic and dynamics of the body fluids in humans, animals and plants. Hemodynamics deals with body fluids in humans. Classical hemodynamics deals with in vivo and in vitro measurements of pressure, flow and resistance. modern biofluid mechanics measures and analyzes local time dependent velocities and flow in blood vessels, the respiratory system, the lymphatic system and the microcirculation [1, 2]. Many authors have reported that rheologic and fluid dynamic property of blood and its flow behavior through non- uniform cross section of the tube could play an important role in the fundamental understanding, diagnosis and treatment of many cardiovascular diseases [25]. Any alteration in natural pressure and velocity of blood can be a sign of a defect. One of the most important diseases which is significant in controlling blood velocity and pressure is known as atherosclerosis. One of the atherosclerosis complications is hypertension. To study the blood flow in arteries, we analyze the constitutive equations like pressure, velocity, flow rate and shear stress can assist in assessment and prevention of related diseases. Banerjee and Back [5] studied blood flow in an artery of dog numerically using computational Fluid dynamics and found pressure difference graph for a periodic pulsatile. In this study, wall vessels and blood flow considered to be rigid and Newtonian respectively. The problem of determining the motion of a liquid in an elastic tube when subjected to a pressure gradient which is a periodic function of the time arises in connection with the flow of blood in the large arteries (Helps and Mc Donald 1954, Womersley 1954). Attempts have been made in the past to measure the rate of flow in the aorta and femoral artery of the dog and rabbit (Shipley, Gregg and Schroder 1943) and to relate to these observations to the varying pressure. In the absence of adequate mathematical theory, these were not very successful. After some time, fair agreements have been shown between the observed rates of flow and simple solution for oscillatory motion of a viscous liquid in a tube with rigid walls.

In this chapter we shall consider a very simplified model of the arterial system, which consists essentially of the laminar flow of viscous, incompressible, Newtonian fluid in an infinitely long, uniform, rigid cylindrical tube. Such a system is characterized in terms of the Navier-Stokes equations. From these general equations, we shall obtain analytical solution of the fluid velocity and flow rate under the prescribed boundary conditions. Finally the relationship between pressure gradient and the time rate of change of pressure will be discussed.

3.2 Oscillatory Flow Model

The equations governing the laminar flow of viscous incompressible fluid, expressed in cylindrical coordinates (see figure 3.1) are given by [20]

The equation of continuity of mass

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{1}{r}\frac{\partial v}{\partial \theta}+\frac{\partial w}{\partial z}=0 \qquad (3.1)$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial \theta}-\frac{v^2}{r}+w\frac{\partial u}{\partial z}\right)=F_r-\frac{\partial p}{\partial r}+\mu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v}{\partial \theta}+\frac{\partial^2 u}{\partial z^2}\right) \qquad (3.2)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \theta}+\frac{u\,v}{r}+w\frac{\partial v}{\partial z}\right)=F_\theta-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u}{\partial \theta}+\frac{\partial^2 v}{\partial z^2}\right) \qquad (3.3)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+\frac{v}{r}\frac{\partial w}{\partial \theta}+w\frac{\partial v}{\partial z}\right)=F_z-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}+\frac{\partial^2 v}{\partial z^2}\right) \qquad (3.4)$$

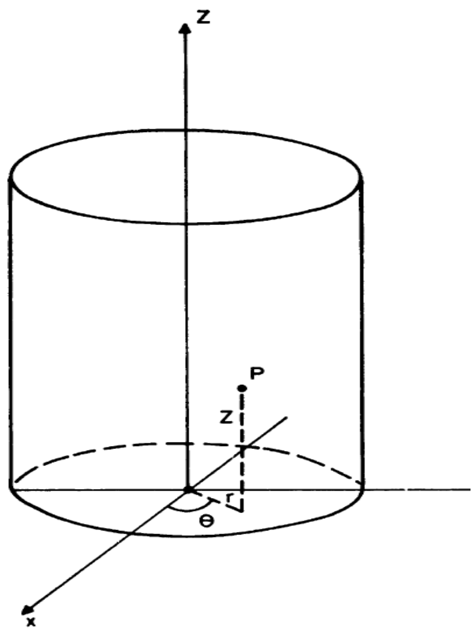

Figure 3.1 Cylindrical Coordinates of a point within the flow along the z axis.

The assumptions made the particular flow process under consideration through a straight, rigid, circular tube are as follows. (See figure 3.2)

(i) Radial and tangential motions of the fluid are neglected, u = 0, v = 0 .

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(ii) The fluid velocity along the axis of the tube (the z axis) is independent of the distance z, $\frac{\partial w}{\partial z} = 0$ that is the value of w remains unchanged along the tube axis.

(iii) w is a function of the radial coordinate r, and time t, that is w = w(r, t).

(iv) The fluid is subjected to a longitudinal periodic pressure gradient

$$-\frac{\partial p}{\partial z} = Ae^{i\omega t} \qquad (3.5)$$

where A is a complex constant denoting the magnitude of the pressure gradient and ω = ft is the phase. The pressure gradient along the radial (r) and tangential (θ) directions are zero.

(v) The body forces F= (F_r, F_θ, F_z) is neglected i.e; $F_r = F_\theta = F_z = 0$.

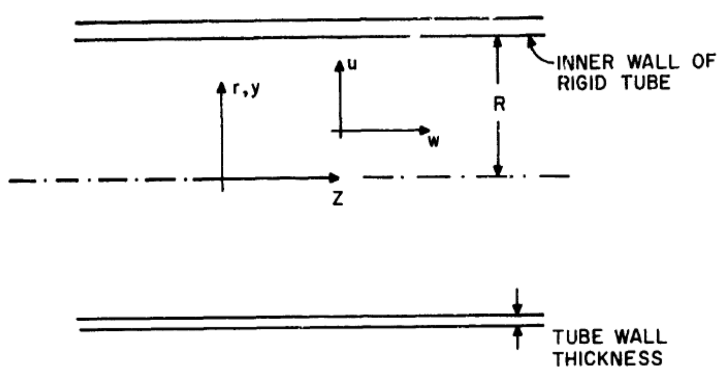

Figure 3.2 Coordinate Flow System

If we impose the restrictions as given in the assumptions (i), (ii), (iii), (v), we find that all the terms in the general flow equations (3.1), (3.2) and (3.3) vanish. We are left with following terms of equation (3.4)

$$\rho\left(\frac{\partial w}{\partial t}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) \qquad (3.6)$$

$$\left(\frac{\partial w}{\partial t}\right) = \frac{A}{\rho}e^{i\omega t} + \vartheta\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) \qquad (3.7)$$

to solve the equation (3.7) we need the boundary conditions,

$$\left.\begin{array}{l} w \text{ is finite}, \ at\ r = 0 \\ w = 0, \quad at \ \ r = R \end{array}\right] \qquad (3.8)$$

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3.3 Solution Of the Problem

Since w is a function of r, and t, therefore we assume the solution of (3.7) as follows:

$$w = g(r)e^{i\omega t} \tag{3.9}$$

$$\frac{\partial w}{\partial t} = i\omega g(r)e^{i\omega t} \tag{3.10}$$

$$\frac{\partial w}{\partial r} = g' e^{i\omega t} \tag{3.11}$$

$$\frac{\partial^2 w}{\partial r^2} = g'' e^{i\omega t} \tag{3.12}$$

substituting the values of (3.9), (3.10), (3.11) and (3.12) into equation (3.7) we get

$$g'' + \frac{1}{r}g' - \frac{i\omega}{\vartheta}g = -\frac{A}{\mu} \tag{3.13}$$

Or

$$\frac{d^2g}{dr^2} + \frac{1}{r}\frac{dg}{dr} - \frac{i\omega}{\vartheta}g = -\frac{A}{\mu} \tag{3.14}$$

Now we write equation (3.14) in terms of a new independent non dimensional variable $y = \frac{r}{R}$. Thus in terms of independent variable y, equation (3.13) has the form

$$\frac{d^2g}{dy^2} + \frac{1}{y}\frac{dg}{dy^2} - \frac{i\omega R^2}{\vartheta}g = -\frac{AR^2}{\mu}$$

$$\frac{d^2g}{dy^2} + \frac{1}{y}\frac{dg}{dy^2} - i\alpha^2 g = -\frac{AR^2}{\mu} \tag{3.15}$$

where $\alpha^2 = \frac{\omega R^2}{\vartheta}$ is dimension less Womersley parameter.

$$\frac{d^2g}{dy^2} + \frac{1}{y}\frac{dg}{dy^2} - \lambda^2 g = -\frac{AR^2}{\mu} \tag{3.16}$$

Equation (3.16) is a non homogeneous modified Bessel's differential equation. The solution of equation (3.16) is

$$g(y) = C_1 I_0(\lambda y) + C_2 K_0(\lambda y) + \frac{AR^2}{\lambda^2 \mu}$$

Or

$$g(y) = C_1 J_0(i\lambda y) + C_2 Y_0(i\lambda y) + \frac{AR^2}{\lambda^2 \mu} \tag{3.17}$$

since g is finite on the y=0, and since $Y_0(0)$ is not finite, then C_2 =0 has to be zero, we have

$$g(y) = C_1 J_0(i\lambda y) + \frac{AR^2}{\lambda^2 \mu} \tag{3.18}$$

Now using another condition, g = 0, at y = 1 and find the value of

$C_1 = -\frac{AR^2}{\lambda^2 \mu}\frac{1}{J_0(i\lambda)} = -\frac{AR^2}{i\alpha^2 \mu J_0\left(i^{\frac{3}{2}}\alpha\right)}$ then from (3.18)

$$g(y) = -\frac{AR^2}{i\alpha^2 \mu J_0\left(i^{\frac{3}{2}}\alpha\right)} J_0\left(i^{\frac{3}{2}}\alpha y\right) + \frac{AR^2}{\mu i \alpha^2} = \frac{AR^2}{\mu i \alpha^2}\left[1 - \frac{J_0\left(i^{\frac{3}{2}}\alpha y\right)}{J_0\left(i^{\frac{3}{2}}\alpha\right)}\right] \tag{3.19}$$

putting the (3.19) into (3.9) we get

w (r, t) = g(r) $e^{i\omega t}$ = w(y, t) = g(y) $e^{i\omega t}$

$$w(y,t) = \frac{AR^2}{\mu i \alpha^2}\left[1 - \frac{J_0\left(i^{\frac{3}{2}}\alpha y\right)}{J_0\left(i^{\frac{3}{2}}\alpha\right)}\right] e^{i\omega t} \tag{3.20}$$

3.4 Limiting Form of the Solution

For fixed values of R and ν, the value of α varies as $(\omega)^{\frac{1}{2}}$ in $\alpha = \left(\frac{R^2\omega}{\nu}\right)^{\frac{1}{2}}$. So it is necessary to look at the variation of the fluid velocity w for small and large values of α. but here we shall discuss only small values of α.

If we include a phase lag between the oscillatory pressure and the flow generated, then pressure gradient imposed on the fluid has the form

$$-\frac{\partial p}{\partial z} = Me^{i(\omega t - \emptyset)} = M\cos(\omega t - \emptyset) + iM\sin(\omega t - \emptyset) \tag{3.21}$$

instead of the form $-\frac{\partial p}{\partial z} = Ae^{i\omega t}$. Here M is the magnitude of the pressure gradient and $\emptyset$ denote the phase lag of the flow rate behind the pressure gradient. The fluid velocity w as described by equation

$$w = \frac{MR^2}{\mu i \alpha^2}\left[1 - \frac{J_0\left(i^{\frac{3}{2}}\alpha y\right)}{J_0\left(i^{\frac{3}{2}}\alpha\right)}\right][\cos(\omega t - \emptyset) + i\sin(\omega t - \emptyset)] \tag{3.22}$$

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For small values of the fluid parameter α, we have

$$J_n(x) = x^n \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+n} m!\,(m+n)!}$$

For n = 0

$$J_0(x) = 1 - \left(\frac{x}{2}\right)^2 + \frac{(x)^4}{2^2.4^2} - \frac{(x)^6}{2^2.4^2.6^2} + \cdots \ldots \ldots \ldots ..$$

replace x by iy , we get

$$J_0(iy) = 1 - \left(\frac{iy}{2}\right)^2 + \frac{(iy)^4}{2^2.4^2} - \frac{(iy)^6}{2^2.4^2.6^2} + \cdots \ldots \ldots \ldots ..$$

$$J_0(iy) = 1 + \left(\frac{y}{2}\right)^2 + \frac{(y)^4}{2^2.4^2} + \frac{(y)^6}{2^2.4^2.6^2} + \cdots \ldots \ldots \ldots ..$$

$$J_0\left(i^{\frac{3}{2}}\alpha y\right) = 1 - \left(i^{\frac{3}{2}}\alpha\frac{y}{2}\right)^2 + \frac{\left(i^{\frac{3}{2}}\alpha y\right)^4}{2^2.4^2} - \frac{\left(i^{\frac{3}{2}}\alpha y\right)^6}{2^2.4^2.6^2} + \cdots \ldots \ldots \ldots .. \qquad (3.23)$$

$$J_0\left(i^{\frac{3}{2}}\alpha\right) = 1 - \left(i^{\frac{3}{2}}\alpha\frac{1}{2}\right)^2 + \frac{\left(i^{\frac{3}{2}}\alpha\right)^4}{2^2.4^2} - \frac{\left(i^{\frac{3}{2}}\alpha\right)^6}{2^2.4^2.6^2} + \cdots \ldots \ldots \ldots .. \qquad (3.24)$$

Since $0 < y \ll 1, for\ \alpha \ll 3$, we neglecting the higher order term of (3.23) and (3.24) then reducible form is

$$J_0\left(i^{\frac{3}{2}}\alpha y\right) = 1 + \frac{i\alpha^2 y^2}{4} \qquad (3.25)$$

$$J_0\left(i^{\frac{3}{2}}\alpha\right) = 1 + \frac{i\alpha^2}{4} \qquad (3.26)$$

then the term

$$\left[1 - \frac{J_0\left(i^{\frac{3}{2}}\alpha y\right)}{J_0\left(i^{\frac{3}{2}}\alpha\right)}\right] = 1 - \left\{\frac{4 + i\alpha^2 y^2}{4 + i\alpha^2}\right\} = \frac{i\alpha^2(1-y^2)}{4+i\alpha^2} \qquad (3.27)$$

then from (3.22)

$$\frac{MR^2}{\mu i\alpha^2}\left[\frac{i\alpha^2(1-y^2)}{4+i\alpha^2}\right][\cos(\omega t - \emptyset) + i\sin(\omega t - \emptyset)]$$

$$\frac{MR^2}{\mu}\left[\frac{(1-y^2)}{4+i\alpha^2}\right][\cos(\omega t - \emptyset) + i\sin(\omega t - \emptyset)]$$

For small values α, α^2 is neglected from denominator. Then we get

$$w(y,t) = \frac{MR^2}{\mu}\left[\frac{(1-y^2)}{4}\right][\cos(\omega t - \emptyset) + i\sin(\omega t - \emptyset)] \quad (3.28)$$

in equation (3.28), the real part has significance. The imaginary part determines the phase of fluid velocity. Thus for small values of $\alpha,$ the fluid velocity w is a function of y and t is given by

$$w = w(y,t) = \frac{MR^2}{\mu}\left[\frac{(1-y^2)}{4}\right]\cos(\omega t - \emptyset) \quad (3.29)$$

In equation (3.29), if we consider there is no phase lag i.e.; $\emptyset = 0$

and $\omega = 2\pi f = 0$, then $\cos(0-0) = \cos(0) = 1$, then we get

$$w = w(y,t) = \frac{MR^2}{\mu}\left[\frac{(1-y^2)}{4}\right] \quad (3.30)$$

It is clear from (3.30), dependence of the fluid velocity on time t has been eliminated due to $\emptyset = 0$ and $\omega = 0$. Moreover, since

$y = \frac{r}{R} \Longrightarrow 1 - y^2 = 1 - \frac{r^2}{R^2} = \frac{R^2-r^2}{R^2}$, then from (3.30) we get

$$w = w(r) = \frac{MR^2}{4\mu}\left[\frac{R^2-r^2}{R^2}\right] = \frac{M}{4\mu}(R^2 - r^2) \quad (3.31)$$

in this equation fluid velocity w is function of the radial coordinate r only.

We know that the fluid velocity for the Poiseuille flow in a straight rigid tube is given by

$$w = w(r) = \frac{p_1-p_2}{4\mu L}(R^2 - r^2) \quad (3.32)$$

from (3.31) and (3.32)

$$\frac{M}{4\mu}(R^2 - r^2) = \frac{p_1-p_2}{4\mu L}(R^2 - r^2)$$

$M = \frac{p_1-p_2}{L} = \frac{\Delta p}{L}$. Thus equation (3.31) shows that fluid velocity for stationary Poiseuille flow in a straight rigid tube.

3.5 Modified Form of the Solution

we have , the flow velocity of an oscillatory viscous fluid flow from (3.20) is given by

$$\mathrm{w(y,\ t)} = \frac{AR^2}{\mu i\alpha^2}\left[1 - \frac{J_0\left(i^{\frac{3}{2}}\alpha y\right)}{J_0\left(i^{\frac{3}{2}}\alpha\right)}\right]e^{i\omega t} \quad (3.33)$$

This equation will provide velocity profiles as a function of A,R,μ, α, and r. But we see that given equation is difficult for calculation purposes, so we need to modify this equation in terms of modulus and phase form using the (McLahlan,1934),[18] formula as follows:

$J_0\left(i^{\frac{3}{2}}\alpha\right) = M_0(\alpha)e^{i\theta_0\,(\alpha)}$ and $J_0\left(i^{\frac{3}{2}}\alpha y\right) = M_0(\alpha y)e^{i\theta_0\,(\alpha y)}$, using these relations in (3.33) we get

$$w(y, t) = \frac{AR^2}{\mu i \alpha^2}\left[1 - \frac{M_0(\alpha y)e^{i\theta_0\,(\alpha y)}}{M_0(\alpha)e^{i\theta_0\,(\alpha)}}\right]e^{i\omega t}$$

where

$$M_0(\alpha)=\left|J_0\left(i^{\frac{3}{2}}\alpha\right)\right|, \qquad M_0(\alpha y)=\left|J_0\left(i^{\frac{3}{2}}\alpha y\right)\right|$$

$$\theta_0\,(\alpha) = phase\left\{J_0\left(i^{\frac{3}{2}}\alpha\right)\right\}, \quad \theta_0\,(\alpha y) = phase\left\{J_0\left(i^{\frac{3}{2}}\alpha y\right)\right\}$$

$$w(y, t) = \frac{AR^2}{\mu i \alpha^2}\left[1 - \frac{M_0(\alpha y)e^{i(\theta_0\,(\alpha y) - \theta_0\,(\alpha))}}{M_0(\alpha)}\right]e^{i\omega t}$$

$$w(y, t) = \frac{AR^2}{\mu i \alpha^2}\left[1 - h_0\, e^{-i\delta_0}\right]e^{i\omega t}$$

where $h_0 = \frac{M_0(\alpha y)}{M_0(\alpha)}$, and $\delta_0 = \theta_0\,(\alpha) - \theta_0\,(\alpha y)$

$$w(y, t) = \frac{AR^2}{\mu i \alpha^2}[1 - h_0\,(\cos\delta_0 - i\sin\delta_0)]e^{i\omega t}$$

$$w(y, t) = \frac{AR^2}{\mu i \alpha^2}[(1 - h_0\,\cos\delta_0) + (ih_0\,\sin\delta_0)]e^{i\omega t} \tag{3.34}$$

to simplify the equation (3.34), we set

$$M_0' = \sqrt{(h_0\,\sin\delta_0)^2 + (1 - h_0\,\cos\delta_0)^2}$$

$$M_0' = \sqrt{(1 + h_0^2 - 2h_0\,\cos\delta_0)}$$

From figure (3.3), $\tan\varepsilon_0 = \frac{h_0\,\sin\delta_0}{1 - h_0\,\cos\delta_0}$

or

$$M_0'\cos\varepsilon_0 = 1 - h_0\,\cos\delta_0 \tag{3.35}$$

$$M_0'\sin\varepsilon_0 = h_0\,\sin\delta_0 \tag{3.36}$$

multiplying by i, in (3.36) and adding (3.35) and (3.36), we get

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$M_0' \cos \varepsilon_0 + iM_0' \sin \varepsilon_0 = 1 - h_0 \cos \delta_0 + ih_0 \sin \delta_0$ (3.37)

now from (3.34) and (3.37), we have

$$w(y, t) = \frac{AR^2}{\mu i\alpha^2}[M_0' \cos \varepsilon_0 + iM_0' \sin \varepsilon_0]e^{i\omega t} = \frac{AR^2 M_0'}{\mu i\alpha^2} e^{i\varepsilon_0} e^{i\omega t}$$

$$w(y, t) = \frac{AR^2 M_0'}{\mu i\alpha^2} e^{i(\varepsilon_0+\omega t)} e^{i\omega t} \quad (3.38)$$

if there is a phase lag of the flow behind the pressure gradient, then replace factor A $e^{i\omega t}$ in equation (3.38) by M $e^{i(\omega t-\emptyset)}$. Thus from (3.38), we have

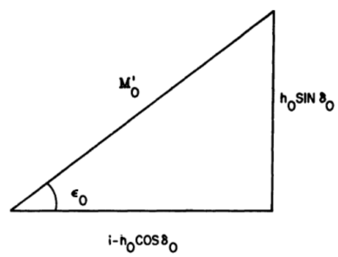

Figure: 3.3 Trigonometric Diagram for velocity

$$w(y, t) = \frac{MR^2 M_0'}{\mu i\alpha^2} e^{i(\varepsilon_0+\omega t-\emptyset)}$$

$$w(y, t) = \frac{MR^2 M_0'}{\mu \alpha^2}\left[\frac{1}{i}\{\cos(\varepsilon_0 + \omega t - \emptyset) + \sin(\varepsilon_0 + \omega t - \emptyset)\}\right] \quad (3.39)$$

The real part of (3.39) describes the actual flow velocity along the tube. Thus

$$w(y, t) = \frac{MR^2 M_0'}{\mu \alpha^2}[\sin(\varepsilon_0 + \omega t - \emptyset)] \quad (3.40)$$

3.6 Volume of Flow Rate

The volume rate of flow Q is obtained by integrating the fluid velocity w = w(y, t) with respect to cross section area S of the tube, that is

$$Q = 2\pi \int_{r=0}^{r=R} w(y,t) r\, dr$$

since $y = \frac{r}{R},\ at\ r = 0,\ \ y = 0,\ and\ at\ r = R,\ \ y = 1$

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$$Q = 2\pi \int_{y=0}^{y=1} w(y,t)(yR)d(yR)$$

$$Q = 2\pi R^2 \int_{y=0}^{y=1} w(y,t)y\, dy = 2\pi R^2 \int_0^1 w(y,t)y\, dy \qquad (3.41)$$

substituting the value of w(y, t) from (3.33) into (3.41), we have

$$Q = 2\pi R^2 \int_0^1 \left[\frac{A}{i\omega\rho} \left\{ 1 - \frac{J_0\left(i^{\frac{3}{2}}\alpha y\right)}{J_0\left(i^{\frac{3}{2}}\alpha\right)} \right\} e^{i\omega t} \right] y\, dy$$, after simplification we get

$$Q(t) = \pi R^2 \left[\frac{A}{i\omega\rho} \left\{ 1 - \frac{2J_1\left(i^{\frac{3}{2}}\alpha\right)}{i^{\frac{3}{2}}\alpha J_0\left(i^{\frac{3}{2}}\alpha\right)} \right\} e^{i\omega t} \right] \qquad (3.42)$$

Now we obtain the average velocity

$$\overline{w} = \overline{w}(t) = \frac{Q(t)}{\pi R^2} \qquad (3.43)$$

where πR^2 is the cross sectional area of the tube.

$$\overline{w} = \overline{w}(t) = \left[\frac{A}{i\omega\rho} \left\{ 1 - \frac{2J_1\left(i^{\frac{3}{2}}\alpha\right)}{i^{\frac{3}{2}}\alpha J_0\left(i^{\frac{3}{2}}\alpha\right)} \right\} e^{i\omega t} \right]$$

$$\overline{w} = \overline{w}(t) = \left[\frac{AR^2}{i\alpha^2\mu} \left\{ 1 - \frac{2J_1\left(i^{\frac{3}{2}}\alpha\right)}{i^{\frac{3}{2}}\alpha J_0\left(i^{\frac{3}{2}}\alpha\right)} \right\} e^{i\omega t} \right] \qquad (3.44)$$

we may write equation (3.44) in modulus and phase form as follows:

$$M_0(\alpha) = \left| J_0\left(i^{\frac{3}{2}}\alpha\right) \right|, \qquad M_1(\alpha) = \left| J_1\left(i^{\frac{3}{2}}\alpha\right) \right|$$

$$\theta_0(\alpha) = phase\left\{ J_0\left(i^{\frac{3}{2}}\alpha\right) \right\}, \quad \theta_1(\alpha) = phase\left\{ J_1\left(i^{\frac{3}{2}}\alpha\right) \right\}$$

$$M'_{10}(\alpha) = \left| 1 - \frac{2J_1\left(i^{\frac{3}{2}}\alpha\right)}{i^{\frac{3}{2}}\alpha J_0\left(i^{\frac{3}{2}}\alpha\right)} \right|$$

$$h'_{10}(\alpha) = \frac{2M_1(\alpha)}{\alpha M_0(\alpha)}$$

$$\delta_{10}(\alpha) = phase\left(i^{\frac{3}{2}}\right) - \theta_1(\alpha) + \theta_0(\alpha) = 135^0 - \theta_1(\alpha) + \theta_0(\alpha)$$

$$\overline{w} = \overline{w}(t) = \left[\frac{AR^2}{i\alpha^2\mu}\left\{1 - \frac{2\left|J_1\left(i^{\frac{3}{2}}\alpha\right)\right|.phase\left\{J_1\left(i^{\frac{3}{2}}\alpha\right)\right\}}{phase\left(i^{\frac{3}{2}}\right)\alpha\left|J_0\left(i^{\frac{3}{2}}\alpha\right)\right|.phase\left\{J_0\left(i^{\frac{3}{2}}\alpha\right)\right\}}\right\}e^{i\omega t}\right]$$

$$\overline{w} = \frac{AR^2}{i\alpha^2\mu}\left[1 - \frac{2M_1(\alpha).e^{i\theta_1(\alpha)}}{\alpha M_0(\alpha).e^{\frac{3\pi i}{4}}.e^{i\theta_0(\alpha)}}\right]e^{i\omega t}$$

$$\overline{w} = \frac{AR^2}{i\alpha^2\mu}\left[1 - \frac{2M_1(\alpha).e^{i\theta_1(\alpha) - 135^0 - \theta_0(\alpha)}}{\alpha M_0(\alpha)}\right]e^{i\omega t}$$

$$\overline{w} = \frac{AR^2}{\mu i\alpha^2}\left[1 - h_{10}(\alpha)e^{-i\delta_{10}}\right]e^{i\omega t}$$

where $h_{10}(\alpha) = \frac{2M_1(\alpha)}{\alpha M_0(\alpha)}$, and $\delta_{10} = \theta_0(\alpha) + 135^0 - \theta_1(\alpha)$

$$\overline{w} = \frac{AR^2}{\mu i\alpha^2}[1 - h_{10}(\alpha)(\cos\delta_{10} - i\sin\delta_{10})]e^{i\omega t}$$

$$\overline{w} = \frac{AR^2}{\mu i\alpha^2}[(1 - h_{10}(\alpha)\cos\delta_{10}) + (ih_{10}(\alpha)\sin\delta_{10})]e^{i\omega t} \quad (3.45)$$

we will solve the equation (3.45) with help of fig. (3.4)

$$M'_{10} = \sqrt{(h_{10}\sin\delta_{10})^2 + (1 - h_{10}\cos\delta_{10})^2}$$

$$M'_{10} = \sqrt{(1 + h_{10}^2 - 2h_{10}\cos\delta_{10})}$$

From figure (3.4), $\tan\varepsilon'_{10} = \frac{h_{10}\sin\delta_{10}}{1 - h_{10}\cos\delta_{10}}$

or

$$M'_{10}\cos\varepsilon'_{10} = 1 - h_{10}\cos\delta_{10} \quad (3.46)$$

$$M'_{10}\sin\varepsilon'_{10} = h_{10}\sin\delta_{10} \quad (3.47)$$

multiplying by i, in (3.47) and adding (3.46) and (3.47), we get

$$M'_{10}\cos\varepsilon'_{10} + iM'_{10}\sin\varepsilon'_{10} = 1 - h_{10}\cos\delta_{10} + ih_{10}\sin\delta_{10} \quad (3.48)$$

now from (3.48) and (3.45), we have

$$\overline{w} = \frac{AR^2}{\mu i\alpha^2}[M'_{10}\cos\varepsilon_{10} + iM'_{10}\sin\varepsilon_{10}]e^{i\omega t} = \frac{AR^2 M'_{10}}{\mu i\alpha^2}e^{i\varepsilon'_{10}}e^{i\omega t}$$

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$$\overline{w} = \frac{AR^2M'_{10}}{\mu i\alpha^2}e^{i(\varepsilon'_{10}+\omega t)} \tag{3.49}$$

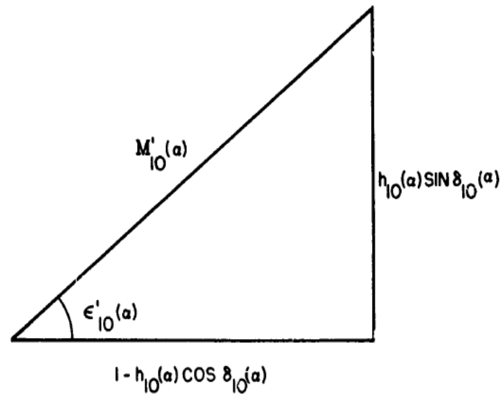

Figure-3.4 Trigonometric Diagram for flow rate

If there is a negative phase lag ∅ between the flow velocity and the applied pressure gradient, then we replace the factor A$e^{i\omega t}$ by M$e^{i(\omega t-\varphi)}$. Thus from equation (3.49), we have

$$\overline{w} = \frac{MR^2M'_{10}}{\mu i\alpha^2}e^{i(\varepsilon'_{10}+\omega t-\varphi)}$$

$$\overline{w} = \frac{MR^2M'_{10}}{\mu i\alpha^2}\{\cos(\varepsilon'_{10}+\omega t-\varphi)+i\sin(\varepsilon'_{10}+\omega t-\varphi)\}$$

$$\overline{w} = \frac{MR^2M'_{10}}{\mu\alpha^2}\left\{\frac{1}{i}\cos(\varepsilon'_{10}+\omega t-\varphi)+\sin(\varepsilon'_{10}+\omega t-\varphi)\right\} \tag{3.50}$$

The actual average flow velocity along the tube axis is given by the real part of the equation (3.50) as

$$\overline{w} = \frac{MR^2M'_{10}}{\mu\alpha^2}\sin(\varepsilon'_{10}+\omega t-\varphi) \tag{3.51}$$

The actual volume flow rate Q to the actual average velocity is given by

$$Q = Q(t) = \overline{w}(t).\pi R^2$$

$$Q = Q(t) = \frac{\pi MR^4M'_{10}}{\mu\alpha^2}\sin(\varepsilon'_{10}+\omega t-\varphi) \tag{3.52}$$

It is clear from (3.52), the maximum value of $\sin(\varepsilon'_{10}+\omega t-\varphi)$ = 1, then we may write

$$Q_{max.}(t) = \frac{\pi MR^4M'_{10}}{\mu\alpha^2} \tag{3.53}$$

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Moreover, we note that the volume flow rate under steady, laminar conditions according to Poiseuille's formula is

$$Q = Q_{steady} = \frac{\pi R^4}{8\mu L}(p_1 - p_2) \tag{3.54}$$

For a comparison of $Q_{max.}(t)$ to Q_{steady}, we have

$$\frac{Q_{max.}(t)}{Q_{steady}} = \frac{8MM'_{10}(\alpha)}{\alpha^2 \frac{(p_1 - p_2)}{L}}$$

if we take $M = \frac{(p_1 - p_2)}{L}$ in Poiseuille's flow then

$$\frac{Q_{max.}(t)}{Q_{steady}} = \frac{8M'_{10}(\alpha)}{\alpha^2} \tag{3.55}$$

3.7 Discussion and Results

In the equation (3.55), the ratio $\frac{Q_{max.}(t)}{Q_{steady}}$ decreases as α increases. For the variation of $M'_{10}(\alpha)$ as function α, we plot the ratio $\frac{8M'_{10}(\alpha)}{\alpha^2}$ against α.

As $\alpha \to 0$, $M'_{10}(\alpha) \to \frac{1}{8}\alpha^2$ so that $\frac{M'_{10}(\alpha)}{\alpha^2} \to \frac{1}{8}$, and the flow at small values of α is the same as given by Poiseuille's formula. The values of the quantities $M'_{10}(\alpha)$ and $\frac{M'_{10}(\alpha)}{\alpha^2}$ are given in the tables I, II and III of (Womersley, 1957), for the values of $\alpha = 0 \; to \; \alpha = 10$ at the intervals of 0.05 in α. But to calculate w(t) and Q(t) for values of the $\alpha > 10$, we may use the asymptotic expansions given by (McLachlan, 1961) of the modulus $M'_{10}(\alpha), and\ the\ phase\ \varepsilon'_{10}$ as

$$M'_{10}(\alpha) = 1 - \frac{\sqrt{2}}{\alpha} + \frac{1}{\alpha^2}$$

$$\varepsilon'_{10}(\alpha) = \frac{\sqrt{2}}{\alpha} + \frac{1}{\alpha^2} + \frac{19}{24\sqrt{2}\,\alpha^3}$$

It is clear from the equation (3.40), the factor α^2 occurs in the denominator and is factor which contributes to the amplitude of the flow velocity. so we can say that as the value of α^2 increases, the amplitude of the fluid velocity decreases, that is the flow profile tends to flatten. The wide variation in the maximum flow rate $Q_{max.}(t)$ for different values of α raises the question: how much α likely to vary in different animals ? The answer will be as follows: The driving pressure is harmonic of frequency, $\omega = 2\pi f$

the diameter of the human femoral artery = 0.5 cm

f = pulse rate = 72 per minute

ν = kinematic viscosity of blood = 0.038 stokes

then the value of α is give by

$$\alpha = \left(\frac{R^2\omega}{\nu}\right)^{\frac{1}{2}} = \frac{0.5}{2}\left(\frac{2\times3.14\times72}{60}\times\frac{1}{0.038}\right)^{\frac{1}{2}} = 3.52$$

The corresponding values of α for the rabbit and cat are as the same magnitude [Womersley, 1957].

This type of flow application is very helpful in Doppler Ultrasound and MRI in case of diseased artery.

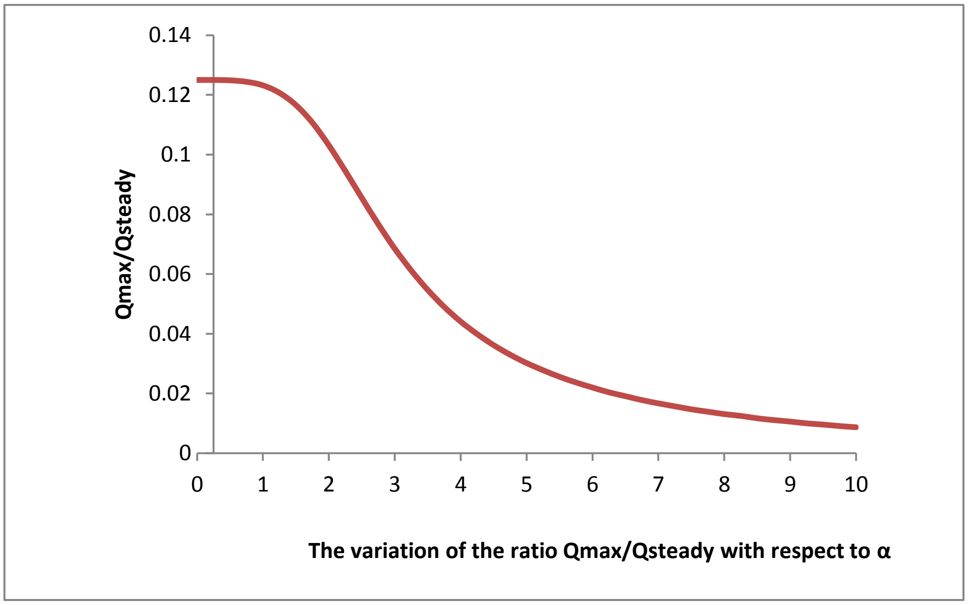

Figure-3.5

References

[1] A.J. Vander, J.H. Sherman, and D.S. Luciano, **Human Physiology: The Mechanism of Body Function**, McGraw-Hill, New York, 1985.

[2] A. Maton. **Human Biology and Health**, Prentice Hall, Englewood Cliffs, New Jersey, 1993.

[3] Alan Jeffery, **Magneto-hydrodynamics**, New York, 1966.

[4] Alizadeh, Seyedpour, S.M, **Calculation and Analysis of Velocity and Viscous Drag in an Artery With Periodic Pressure Gradient**, Chinese Journal of Mechanical Engineering, 2012

[5] Banerjee R.K, Back L.H, **Pulsatile Blood Flow in a Tapered Femoral Artery of a Dog ,Computed and Measured Hydrodynamics**, Journal of Bioengineering Division , p-517-518 , 2001

[6] Fung, Y.C., **Bio-dynamics Circulation**, Springer Verleg, New York, 1984.

[7] Fung, Y.C., **Biomechanics: Motion, Flow**, **Stress and Growth**, Springer Verleg, New York, 1990.

[8] Jagan, N., Mazumdar, **Bio-fluid Mechanics**, World Scientific Press, New Jersey,1992.

[9] J., Mazumdar, **An Introduction to Mathematical Physiology and Biology**, Cambridge University press, 1989.

[10] J. Kneer, J. Sneyd, **Mathematical physiology**, New York, Springer, 1998.

[11] J.T. Ottesen, M.S. Olufsen, J.K. Larsen, **Applied Mathematical Models in Human Physiology**, 2004

[12] Kapur, J.N., **Mathematical Models in Biology and Medicine**, Affiliated East- West Press Ltd., India, 1985.

[13] Lebedev, N.N., **Special Functions and Their Applications**, Prentice Hall, Englewood Cliffs, New Jersey, 1965.

[14] Martini, F.H., **Fundamentals of Anatomy and Physiology**, Prentice Hall, Englewood Cliffs, New Jersey, 1995.

[15] McDonald, D.A., **Blood Flow in Arteries**, Williams, Baltimore, 1960.

[16] McDonald, D.A., **The Relation of Pulsatile Pressure to Flow in Arteries**, J. Physiol., vol. 127, p -533, 1955.

[17] McDonald, D.A., and M.G., Taylor, **The Phase Velocities of Harmonic Components of the Pulse Wave**, J. Physiol., vol. 137, p -87, 1957.

[18] Mclachlan, N.W., **Bessel Functions for Engineers**, Oxford University Press, 1961.

[19] Page R., Painter, Partic, Eden., **Pulsatile Blood Flow, Shear Force, Energy Dissipation and Murray's Law**, J. Theoretical Biology and Modeling, vol. 3- 31, 2006.

[20] Schlichting, H., **Boundary Layer Theory**, McGraw-Hill, 1960.

[21] Thurston, R. and C.E. Martin, Jr., **Periodic Fluid Flow through Circular Orifices**, J. Acoustic. Soc. Am., Vol. 25, p-26, 1953.

[22] Womersley, J.R., **Flow in the Large Arteries and Its Relations to the Oscillating Pressure**, J.Physiol., Vol., 124, p-31-32, 1954.

[23] Womersley, J.R., **Method for Calculation of Velocity, Rate of Flow and Viscous Drag in Arteries when the Pressure Gradient is Known,** J.Physiol., vol., 127, p-553-563, 1955.

[24] Womersley, J.R., **Oscillatory motion of a Viscous Liquid in a Thin Walled Elastic Tube,** Phil. Mag., Vol., 46, p-178-187, 1957.

[25] W.R. Milnor, **Cardiovascular Physiology**, Oxford Press, New York, 1990.

الملـخـص العربي

المشروع البحثي الرئيسي الحالي بعنوان **تدفق الدم في شريان فخذي كبير،** وقد امتد في أكثر من ثلاثة فصول. الفصل ١، ويعطي المعلومات والتنمية الشاملة لميكانيكا الموائع الحيوية . هذا الفصل يتناول المزيد من التعاريف الأساسية ، و معادلات تحكم نماذج مختلفة ، وأنواع مختلفة من السوائل ومجموعة من المعادلات الهيدروديناميكية . في هذا الفصل ناقشنا أيضا أهمية الموضوع ، ونظام القلب والأوعية الدموية، وتوزيع حجم الدم والفرق بين علم التشريح وعلم وظائف الأعضاء.

ويستند الهدف من الفصل ٢ على اهتمام الأبحاث اليوم في ديناميكيات السوائل الحيوية ، مكرسة أكثر وأكثر لنمذجة دقيقه لخصائص تدفق الدم . أصبح واضحا أن الدم هو السائل المعقد ، مع خصائص تبعا لعوامل كثيرة ، لا تقتصر على القص ونسبة الهيماتوكريت . في هذا الفصل ناقشنا في تفاصيل حول ريولوجيا الدم والأوعية الدموية لأنه من المهم جدا أن نفهم تدفق الدم في الشريان . كما تم مناقشة بعضاً من خصائص تدفق الدم في هذا الفصل .

الفصل ٣، ويعرض نموذج نظري من تدفق الدم المتذبذب في أنبوب جامد في الدورة الدموية . و يعتبر الافتراضات من الريولوجيا النيوتونية ويوصف تدفق الدم عن طريق معادلات نافيير ستوكس مع حد تدرج الضغط والشروط الحدية المحددة . في هذا الفصل لدينا الحل التحليلي للمشكلة من حيث السرعة ومعدل التدفق . الهدف الرئيسي من هذا الفصل هو، كيف يمكن لديناميكا الموائع أن تلعب دورا هاما في فهم الجهاز الدوري في الإنسان . هذا النوع من مشاكل تدفق الدم يحدث في جسم الإنسان في حالة الشرايين المريضة.

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